E. Smith

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March. 1831.

CAMBRIDGE

PROBLEMS.

1811—1820.

CAMBRIDGE

PROBLEMS:

BEING

A

COLLECTION OF THE PRINTED QUESTIONS

PROPOSED TO THE CANDIDATES

FOR

THE DEGREE OF BACHELOR OF ARTS

AT THE

GENERAL EXAMINATIONS

FROM 1811 TO 1820 INCLUSIVE.

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ADVERTISEMENT.

For the accommodation of those who are in possession of the first series of the Cambridge Problems ending 1810, the Publishers have printed a number of copies of the second series, from 1811 to 1820; which may be had independently of the first series.

Cambridge, Oct. 20, 1820.

UNIVERSITY OF CAMBRIDGE,

1820.

A LIST of the MODERATORS from the year 1810 to the present time: with a Summary of the ACADEMICAL HONOURS obtained during that period.

	Wranglers.	Sen^r Opt.	Jun^r Opt.	Total.
1811.				
Thomas Jephson, M. A. Joh. T. Turton, M. A. Cath. Hall.	15	17	11	43
1812.				
T. Turton, M. A. Cath. Hall. J. D. Hustler, M. A. Trin. Coll.	17	18	8	43
1813.				
Thomas Jephson, M. A. Joh. G. Macfarlan, M. A. Trin. Coll.	15	14	15	44
1814.				
G. Macfarlan, M. A. Trin. Coll. Miles Bland, M. A. Joh.	19	17	13	49
1815.				
Miles Bland, M. A. Joh. William Hustler, M. A. Jesus	22	23	9	54
1816.				
Miles Bland, M. A. Joh. William French, M. A. Pemb.	19	15	14	48
1817.				
John White, M. A. Caius G. Peacock, M. A. Trin. Coll.	18	18	12	48

	Wranglers.	Sen[r] Opt.	Jun[r] Opt.	Total.
1818.				
Fearon Fallows, M. A. Joh. William French, M. A. Pemb.	28	30	11	69
1819.				
Richard Gwatkin, M. A. Joh. G. Peacock, M. A. Trin. Coll.	19	23	14	56
1820.				
Henry Wilkinson, M. A. Joh. W. Whewell, M. A. Trin. Coll.	18	19	15	52

CAMBRIDGE PROBLEMS.

1811.

Morning Problems.—Mr. Turton.

Monday, January 14, 1811.

1. The interior angles of a rectilinear figure are in arithmetic progression; the least angle is 120°, and the common difference 5°. Required the number of sides.

2. Given the radii of two spheres, and the line joining their centers; find, in that line, the position of an eye, to which the apparent surfaces will together be the greatest possible.

3. The weight of a globe in air $= W$, and in water $= w$; find its diameter and specific gravity, having given the specific gravity of water (S) and of air (s).

4. Having given the latitude of the place, the day and hour, also the latitude and longitude of a star; find its altitude and azimuth, the point where its vertical circle cuts the ecliptic, and the angle which they make.

5. Find the ratio of the velocity at the extremity of the latus rectum of an ellipse (the force being in the focus) to the velocity in a circle whose radius is

the distance of the nearer apside from the focus; and shew that, as the excentricity is increased, this ratio approaches to a ratio of equality.

6. Shew that the spaces described by a body, impelled from rest by a finite variable force, are, "ipso motûs initio," in the duplicate ratio of the times.

7. If, to the radius unity, A = the sum of the tangents of any number of arcs; B = the sum of the products of every two of them; C = the sum of the products of every three; and so on: shew that the tangent of the sum of those arcs will be

$$\frac{A - C + E - G + \&c.}{1 - B + D - F + \&c.}$$

8. Shew that the fluent of

$$\frac{d\dot{z}}{z\sqrt{(a+bz^n)}} = \frac{2d}{n\sqrt{a}} \times \text{hyp. log.} \frac{\sqrt{(a+bz^n)} - \sqrt{a}}{\sqrt{(bz^n)}};$$

and find the fluent of

$$\frac{z^{\frac{1}{2}n-1}\dot{z}}{z^{pn} \times (a+bz^n)}, \text{ and of } \frac{\dot{z}}{z^{\frac{1}{2}n+1} \times \sqrt{(a+bz^n)}}.$$

9. Find the value of $a.(a+r).(a+2r)$ &c. continued to any number of factors.

10. Find the nature and length of the caustic, when the reflecting curve is a circular arc, and the focus of incident rays is in the circumference of the circle.

11. At a given place, at a given hour, and on a given day, required the point of the compass on which a rainbow would appear.

12. Given the latitude of the place, and the day of the year; find the hour at which two stars, whose right ascensions and declinations are known, will be on the same azimuth.

13. Given the perihelion distance of a comet describing a parabola, and the radius of the earth's orbit, here supposed to be circular; compare the time of the comet's moving through 90 degrees of true anomaly with the length of the solar year.

14. Define the center of spontaneous rotation of a system; explain the principle on which that center may be found; and shew that if the system revolve round an axis, passing through that center perpendicular to the plane of revolution, the former point of impact will become the center of percussion.

Monday Afternoon.—Mr. Turton.

FIFTH AND SIXTH CLASSES.

1. The interest of £25. for $3\frac{1}{4}$ years, at simple interest, was found to be £3. 18*s.* 9*d.*; required the rate per cent. per annum.

2. If the first of six magnitudes be to the second as the third to the fourth, and the fifth to the second as the sixth to the fourth; prove that the first and fifth together will be to the second, as the third and sixth together to the fourth.

3. Given $\dfrac{\sqrt{(a^2+x^2)}}{\sqrt{(a+x)}}$, a minimum; find the value of x.

4. Find the fluent of

$$\frac{d\dot{z}}{\sqrt{(a+bz)}}, \text{ of } \frac{d\dot{z}}{\sqrt{(a+bz^2)}}, \text{ and of } \frac{x\dot{x}}{\sqrt{(ax-x^2)}}.$$

5. Find an expression for the sum of n terms of the series $\frac{1}{5} - \frac{2}{15} + \frac{4}{45}$ &c., that may be applied according as n is an even or an odd number.

6. Shew, that if any momenta be communicated to the parts of a system, its center of gravity will move in the same manner that a body, equal to the sum of the bodies in the system, would move, were it placed in that center, and the same momenta, in the same directions, communicated to it.

7. Compare the time of oscillation in a given cycloid with the time of falling down a vertical line equal to the whole length of the cycloid.

8. Required the equation of which the roots are $\pm\sqrt{(-2)}, 3, 4$.

9. If a body fall through a finite altitude AS, the force varying inversely as the square of the distance, and on AS, a semi-circle ADS be described; prove that the area described by the indefinite radius SD is equal to the area uniformly described in the same time, in a circle whose radius is the half of SA.

10. Given the latitude of the place, and the sun's declination; find the length of the day.

11. Compare the time of descent through any space AS, the force at S varying inversely as the square of the distance, with the periodic time in a circle whose radius is SA.

12. Explain by what means the accelerating forces of bodies are compared; also, by what means their moving forces; and shew that the accelerating force varies as the moving force directly, and the quantity of matter moved inversely.

13. Prove, that if the object placed before a spherical reflector be a straight line, the image is a conic section.

14. Two weights, of which one (P) is known, are connected by a string passing over a fixed pulley; P, in descending from rest through the space s, acquires the velocity a. Find the other weight.

15. Find the variation of the force by which a body describes a parabola, round a center of force in the focus.

16. Find the actual periodic time in a given ellipse, described round a center of force in the focus; supposing that the force at a given distance (d) is to the force of gravity as F to 1.

Mr. Jephson.

THIRD AND FOURTH CLASSES.

1. What is the interest of £115. for $5\frac{1}{2}$ years, at $4\frac{1}{2}$ per cent.?

2. Given $\left.\begin{aligned} x+y-\frac{\sqrt{(x+y)}}{\sqrt{(x-y)}} &= \frac{6}{x-y} \\ x^2+y^2 &= 41 \end{aligned}\right\}$; find x and y; and find x in the equation $a^x+\dfrac{1}{a^x}=b$.

3. Investigate the rule for extracting the square root of a binomial surd, and apply it to determine the square root of $-2m\sqrt{(-m^2)}$.

4. Investigate the fluxional expression for the area of curves, and apply it to find the area that lies between the asymptotes of a common hyperbola.

5. If (a) be an arithmetic, (b) a geometric, (c) a harmonic mean; shew that (a) is greater than (b), and (b) greater than (c).

6. If any quantities, whose differences are inconsiderable in respect to the quantities themselves, be in arithmetical progression, the same quantities are also in geometrical progression.

7. Inscribe, in a triangle, a parallelogram similar to a given parallelogram.

8. Two balls A and B are placed on a billiard-table; in what direction must A, which is perfectly elastic, be struck, that it may hit B after impinging upon two adjacent sides of the table?

9. The specific gravity of a cylinder of known length is greater than that of the fluid in which it is placed; determine the depth of its lower surface.

10. The sine of incidence is to the sine of refraction, out of a denser medium into a rarer, as (n) to (m); give a geometrical construction for determining the greatest possible angle of incidence.

11. Find the fluents of

$$\frac{\dot{x}}{x^3\sqrt{(a^4+x^4)}}, \quad \text{and } \dot{x}\sqrt{(bx-cx^2)}.$$

12. Sum the following series:

$$\frac{1}{2.4.6} + \frac{1}{4.6.8} + \frac{1}{6.8.10} + \text{\&c. in inf.}$$

$$\frac{1}{1.3} - \frac{1}{2.4} + \frac{1}{3.5} - \text{\&c. in inf.}$$

13. The angles at the base of an isosceles spherical triangle are equal.

14. Construct a horizontal dial.

15. If the moveable orbit (NEWT. Sect. IX.) be a logarithmic spiral, what is the nature of the curve traced out in fixed space?

16. Prove that one of the roots of the equation $x^3 - qx - r = 0$, when squared, will lie between (q) and $\left(\frac{q}{3}\right)$.

Evening Problems.—Mr. JEPHSON.

1. Shew that $a^n - b^n$ is, and that $a^n + b^n$ is not, divisible by $a - b$. Is $a^n - b^n$ divisible by $a + b$ (n an integer)?

2. The chord is (ultimately) parallel to the tangent of the middle point of the arc. Required a proof.

3. Prove that the total number of combinations of (n) things is $2^n - 1$; and apply the expression to find the number of different sums that may be formed with a guinea, a half-guinea, a crown, a half-crown, a shilling, and a sixpence.

4. Define similar curves when referred to a point, and shew in what case epicycloids are similar.

5. A lever, at whose extremities P and Q hung by threads, balance each other, is made to revolve about its fulcrum; shew that, if the threads be equal, P and Q describe concentric circles; if unequal, similar segments of circles.

6. Two equal weights balance each other by means of three tacks forming an isosceles triangle, the base of which is horizontal; shew that the whole pressure on the tacks, estimated in the direction in which the weights act, is equal to the sum of the weights.

7. Shew that the pressure of a cylinder, infinite in height upon the earth at rest, equals the weight of another cylinder of the same base, whose length is equal to the radius of the earth.

8. Compare *geometrically* the resistance upon a paraboloid moving in the direction of its axis, with the resistance upon its circumscribing cylinder.

9. If two canals be cut through the earth at rest, then the times of two bodies being attracted through these canals will be equal.

10. $x^y = y^x$; give a geometrical construction for determining two corresponding values of x and y.

11. (1.) Take the fluxion of the two quantities

$$z^{y^z},\ x^{y^z}.$$

(2.) Find the fluents of $\dfrac{\dot{x}}{(a^n+x^n)^{\frac{n+1}{n}}}$,

$$\frac{\dot{x}}{(x-1)^{\frac{3}{2}}.(x+1)^{\frac{1}{2}}},\quad \dot{x}\int \dot{x}\int\frac{\dot{x}}{x}.$$

(3.) Shew that $e^{\int \frac{\dot{\theta}}{\sin. \theta}} = \tan. \left(\frac{\theta}{2}\right)$; (e) being the base of the hyp. logarithms.

12. Sum the following series:

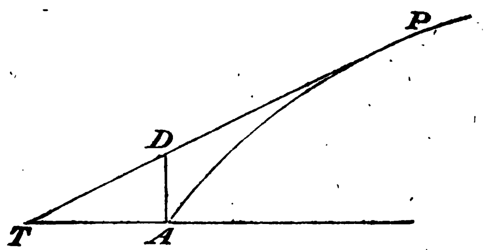
$$\frac{1}{m.(m+r)} + \frac{1}{(m+r).(m+2r)} + \frac{1}{(m+2r).(m+3r)}$$
$+$ &c. in inf.

$$\frac{5}{1.2.3} + \frac{7}{2.3.4} + \frac{9}{3.4.5} + \text{\&c. to } n \text{ terms.}$$

$$1 + 2.3 + 3.3^2 + 4.3^3 + \text{\&c. to } (n) \text{ terms.}$$

13 Divide a given arc (A) less than a quadrant, into two such parts P and Q, that $(\tan. P)^m \times (\tan. Q)^n$ may be a maximum.

14. PT is a tangent to the curve AP, AD is



perpendicular to the axis; find the nature of the curve when $AT \propto (AD)^m$.

15. A known weight (P) at the extremity of a rod which passes through two small rings fixed in the same vertical line, by its pressure puts a solid inclined plane in motion along an horizontal table; given the weight of the plane, find its elevation so that the

velocity communicated to it in a given time may be a maximum.

16. $\{\text{Cos.}\, A \pm \sqrt{(-1)}.\text{sin.}\, A\}^m = \text{cos.}\, m\, A \pm \sqrt{(-1)}.\text{sin.}\, m\, A$. Required proof.

17. A paraboloid of given dimensions is filled with fluid, and placed with its axis parallel to the horizon; how long will the fluid be in running out of it, through a given orifice in the lowest point of the paraboloid?

18. Let a cylinder begin to move from a horizontal position round one of its ends, which remains fixed upon a fulcrum; compare the pressure on that end at the beginning of the motion, with the whole weight of the cylinder.

19. Two bodies are projected at the same time with velocities v and v' from the two extremities of a vertical line; prove, *geometrically*, that if they meet in the middle point of the line, $v \backsim v'$ equals the velocity acquired in the time of meeting.

20. Given the latitude of the place, and the sun's declination; find the time of the day when the hour-angle from noon, and the sun's azimuth from the south, are equal.

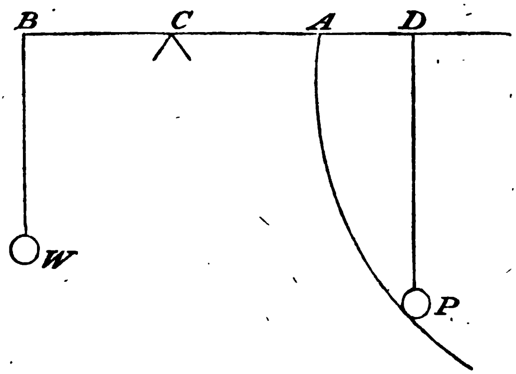
21. Construct the spiral, whose arc is the measure of the ratio between the ordinates which intercept it.

22. If the cycloid described by a body in a resisting medium be formed into a straight line, and ordinates be drawn, which are to the resistances as the length of the pendulum to the weight of the body; then

the area of the curve so traced out is equal to half the length of the cycloid multiplied into the difference between the descent and ascent. NEWT. Vol. II. Prop. xxx. Sect. 6.

23. When a ray of light is incident obliquely upon a spherical reflector, the longitudinal aberration ultimately varies as the versed sine of the arc of the reflector. Required a demonstration.

24. $z^2 = 2ax + x^2$ is the equation to the catenary AP; $AC = CB = (a)$; prove that a weight at B will balance a weight equal to itself made to hang verti-



cally from any point in the axis as D, and pressing upon the catenary; C being the fulcrum of the lever.

25. A cone of given weight and dimensions is placed with its axis horizontal; a known weight (P) is attached to a string, which is wound round its base; find the velocity acquired by P at the end of t''.

Tuesday Morning.—Mr. Jephson.

FIRST AND SECOND CLASSES.

1. Divide (a) into three parts x, y, z, such that $x^m . y^n . z^r$ may be a maximum.

2. From a solid cylinder of given dimensions, cut out a rectangular beam which shall be the strongest possible.

3. Deduce a fluxional expression for the time of emptying vessels through small orifices, and apply it to compare the times of emptying two equal paraboloids; the orifice of the one being in the vertex, and of the other in the base.

4. The focal lengths of a convex and concave lens of different substances, which when united produce images free from colour, are proportional to the dispersing powers of the two mediums. Required a demonstration.

5. Let the weight of a wheel and axle be (w), and let the axis be horizontal; having given a weight (q) applied to the circumference of the axle, and (p) applied to the circumference of the wheel, it is required to find the velocity of the descending weight (p) at the end of t''.

6. Given the sum of the series

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \&c.;$$

find the sum of the series

$$\frac{1}{1^2 . 2^2} + \frac{1}{2^2 . 3^2} + \frac{1}{3^2 . 4^2} + \&c. \text{ in inf.}$$

7. Let the roots of the equation $x^n - px^{n-1} + qx^{n-2} - \&c. \ldots - Qx + R = 0$ be (a), (b), (c), (d), &c. It is required to find the value of

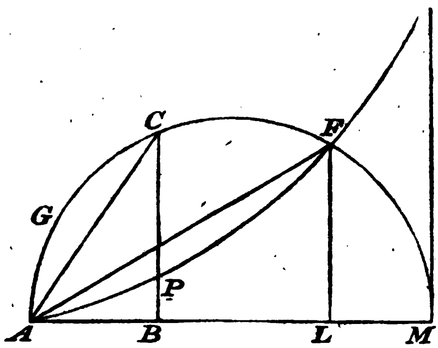
$$\frac{a}{b}+\frac{b}{a}+\frac{a}{c}+\frac{c}{a}+\&c.+\frac{b}{c}+\frac{c}{b}+\&c.+\&c.$$

8. Integrate the fluxional equation

$$\frac{x\dot{y}-y\dot{x}}{x^2+y^2}=K.\dot{\theta}.$$

9. The error in the altitude of a heavenly body is to the corresponding error in right ascension as the sine of the azimuth to the secant of latitude. Required proof.

10. *ACM* is a semi-circle; *BC*, *FL* are ordinates; the area *ABP* is always taken equal to the segment *ACG*; find the equation to the curve traced out by



P; determine the point *F*, in which the curve cuts the semi-circle, and prove that that part of the area which is without the semi-circle is equal to the rectilineal triangle *AFL*.

11. *A* and *B* begin to fall at the same time from

different points in the same vertical line, and with the velocities acquired move along a horizontal plane, till one overtakes the other; shew that the time of *A*'s descent is equal to the time of *B*'s uniform motion.

12. A bag contains (n) balls, *A, B, C, D*, &c.; (p) of them are to be taken out; what is the chance of drawing (p) specified balls?

13. The equation to a catenary is $z^2 = 2ax + x^2$; a parabola, whose latus rectum $= (8a)$ is described with the same vertex and axis; shew that any ordinate of the catenary, together with its corresponding arc, is equal to the corresponding arc of the parabola.

14. A chain whose length is (l) is placed along an inclined plane whose height is (n) and length (m), so that one end may coincide with the lowest point of the plane; shew that the whole time of the chain's sliding off the plane is equal to

$$\sqrt{\frac{(m.l)}{(m-n)}} \times \text{hyp. log.} \left\{\frac{m+\sqrt{(m^2-n^2)}}{n}\right\} \times g; \ (g = 32\tfrac{1}{6}).$$

Tuesday Afternoon.—Mr. Turton.

THIRD AND FOURTH CLASSES.

1. Explain what is meant by the present worth of money due after a certain time; explain also the principle on which is founded the rule for calculating present worth; and find the present worth of £430. due nine months hence, discount being allowed at 4½ per cent. per annum.

2. Solve the equation $3x^n \sqrt[3]{(x^n)} + \frac{4x^n}{\sqrt[3]{(x^n)}} = 4.$

3. Find the fluent of

$$\frac{x^2\dot{x}}{\sqrt{(2ax-x^2)}}, \text{ and that of } \frac{\dot{x} \times (2ax-x)^{\frac{3}{2}}}{x}.$$

4. Find the dimensions of the greatest cylinder that can be cut out of a solid formed by the revolution of a curve round its axis, of which the equation is $a^m x^n = y^{m+n}$, and the whole axis $= b$.

5. Given the altitude of an orifice in the side of a vessel filled with fluid, and the distance on the horizontal plane at which the fluid falls; determine, by construction, the altitude of the vessel.

6. If $a =$ the altitude of a conic frustum, and b, c be the radii of its bases, also $p = 3.14159$ &c.; then will the solidity $= \frac{pa}{3} \times (b^2 + bc + c^2)$. Required a proof.

7. The force varying inversely as $(\text{dist.})^{n+1}$, find the area $ABFD$ (NEWTON, Prop. xxxix.) when the ordinate $= M$ at the distance d from the center; also the *fluxion* of the area $ATVME$, the ordinate of this curve at the distance r from the center being $= N$.

8. Having given the latitude of the place, the sun's declination and altitude; find his azimuth, the time of observation, and the angle of position.

9. Having measured the shadow of a tower on the horizontal plane, on a given day, at noon, in a known

latitude; shew how the altitude of the tower may be found.

10. Construct Newton's Telescope, and investigate its magnifying power.

11. Find the variation of the force tending to the focus of an hyperbola, by which the opposite hyperbola may be described.

12. Supposing the attraction of the earth and moon to be as their quantities of matter directly, and the squares of their distances inversely; having given their quantities of matter and their distance, find that point between them, at which a body would be at rest.

13. Make a body oscillate in a given epicycloid.

Mr. Jephson.

FIFTH AND SIXTH CLASSES.

1. Agreed for the carriage of $2\frac{1}{2}$ tons of goods, $2\frac{2}{10}$ mile, for $\frac{3}{40}$ of a guinea; what is that per cwt. for a mile?

2. What is the amount of £120. 10*s*. for $2\frac{1}{2}$ years, at $4\frac{3}{4}$ per cent.?

3, Divide .7584 by 316.; and find the sum of the recurring decimal 5.72323 &c.

4. Solve the following equations:

$$\left.\begin{aligned} x+y+\sqrt{(x+y)} &= 6 \\ x^2+y^2 &= 10 \end{aligned}\right\}$$

$$\left.\frac{x}{x+4}+\frac{4}{\sqrt{(x+4)}}=\frac{21}{x}\right\}.$$

5. Prove that if $A \propto B$, $A \pm B \propto B$.

6. Divide $x^3 - nax^2 + na^2x - a^3$ by $x - a$.

7. The number of combinations which (n) things admit of when taken four and four together, is to the number which they admit of when taken two and two, as 15 to 2. Required the number of quantities.

8. Define the sine of an arc; and prove that the sines are the same, drawn from either extremity of the arc.

9. Given the center of a circle; find its diameter by means of the compasses alone.

10. Take the fluxion of $(a^2 + x^2) . \sqrt{(a^2 - x^2)}$; and find the $\int \frac{a^2 \dot{x}}{b^4 + c^2 x^2}$.

11. $\frac{x^{n+1}}{2a^n} + \frac{a^n}{2x^{n-1}}$ = a maximum; find x.

12. The sum of (n) terms of the series 1, 3, 5, 7, &c. is to the sum of ($n - 1$) terms of the series 2, 4, 6, &c. as (n) to ($n - 1$). Required proof.

13. Let a perfectly hard body (A) impinge upon another (B) four times as great, and at rest, with a known velocity; find B's velocity after impact.

14. An eye is placed in the principal focus of a concave spherical reflector; compare the apparent magnitudes of the object and image, when the former is situated half way between the focus and surface.

15. An inverted conc is filled with a fluid; deter-

mine at what distance from the vertex a horizontal section will sustain the greatest pressure.

16. When the center of force is without the circle, find its variation in terms of the variable distance.

17. Let a body begin to fall from an infinite distance, force varying as $\frac{1}{(\text{dist.})^2}$: shew that its velocity at any point of its descent is equal to the velocity that it would acquire through the remaining distance, force at that point being continued constant.

Evening Problems.—Mr. TURTON.

1. Define similar curves when referred to their axes; and prove that all parabolas are similar curves.

2. A given rectangular parallelogram is immersed vertically in a fluid, with one side coincident with the surface. From one of its angles, it is required to draw a straight line to the base, so that the pressures on the parts into which the parallelogram is divided may be in a given ratio.

3. Find that point, in the periphery of an ellipse, from which a body must fall, towards the center of force in the focus, through the greatest or least space to acquire the velocity in the curve, at the point from which it fell; and shew, from the fluxional

equation, whether the point determined gives the space a maximum or minimum.

4. Construct GREGORIE's Telescope, and investigate its magnifying power.

5. The times of falling, from different altitudes, into the same center of force, vary as the n^{th} powers of those altitudes. Required the variation of the force.

6. Suppose a person to stand before a vertical plane mirror, at any distance from it; given the altitude of the eye above the bottom of the mirror, find the part of the body that will be seen; and shew that exactly the same part will be visible at all distances from the mirror.

7. If parallel rays be incident on a sphere of given refracting power, find that ray, of which when produced, the part included by the sphere will be to the part included of the refracted ray, in a given ratio.

8. On a given day, at a given hour, and in a given latitude, it is required to find the length and direction of the shadow of a stick of given length, inclined to the horizon at a given angle, and in a given direction.

9. Resolve $\dfrac{1}{1-2lz+z^{2n}}$ into quadratic factors, when l is less than unity.

10. In the expansion of $(a+b+c+d+\&c.)^m$, investigate the coefficient of the term involving the literal product $a^p\,b^q\,c^r\,d^s$ &c.

11. Given $a\dot{x}+(b\dot{x}+cy).\dot{x}=d\dot{y}+(bx+cy).n\dot{x}$; find the relation between x and y.

12. Sum the following series: viz.

$$\frac{3}{1^2.2^2}+\frac{5}{2^2.3^2}+\frac{7}{3^2.4^2}+\text{ \&c. to } n \text{ terms,}$$

and ad inf.

$$\frac{1}{1.2.3.4.5}+\frac{4}{4.5.6.7.8}+\frac{9}{7.8.9.10.11}+\text{ \&c.}$$

to n terms, and ad inf.

13. Find the fluents of

$$\dot{x}\int\frac{\dot{x}}{1-x^3}, \text{ and of } \frac{z^{2n-1}\dot{z}}{(g+hz^n)\sqrt{(e+fz^n)}}.$$

14. Given the latitude and longitude of a fixed star; required the angle which the direction of the earth's way makes with a line drawn from the earth to the star, on a given day.

15. Find the time of the year at which a star, whose right ascension and declination are known, rises with the sun, to an observer in a given latitude.

16. Shew that the force of resistance on a sphere, moving in a fluid, with a given velocity, is to the force that would destroy the sphere's whole motion, in the time in which it would uniformly describe $\frac{8}{3}$ of its diameter, as the density of the fluid to that of the sphere.

17. Find the value of the disturbing force of S upon P, in the case of the three bodies (Sect. XI.); and deduce the mean quantity of that force, during

one revolution of P round T; supposing that $M=$ the force of S at the distance r.

18. If the density of a fluid be proportional to the compressing force, and its particles be attracted by a force varying inversely as the distance from the center; shew that, distances from the center being taken in geometric progression, the corresponding densities will be in geometric progression.

19. Investigate COTES's method of determining the length of an arc of the meridian, on the planisphere.

20. Shew that, in any position of the moon's nodes, the mean horary motion of the nodes, in one synodic revolution of the moon, is equal to half their horary motion when the moon is in syzygies.

21. Having given two distances of a comet, in its parabolic orbit, from the sun, and the angle included; deduce this proportion for determining the perihelion (which is here supposed to lie between those distances.)—The sum of the square roots of the distances is to the difference as the co-tangent of the semi-sum of half the true anomalies to the tangent of the semi-difference of the same.

22. Apply NEWTON's general proposition, in the 8th Section, to the case of COTES's three last spirals; point out the circumstances that determine the spiral; and deduce his construction of the elliptic spiral.

23. The major and minor axes of an ellipse are

given; and a body begins to descend, from the extremity of the minor axis, towards the center of force in the focus, with the velocity in the curve at that point. Compare the time of descent to the focus with the time of revolving in the curve, from the same point to the nearer apside.

24. Suppose a sphere to move in a resisting medium; it is required to cut off a segment, by a plane perpendicular to the direction of its motion, so that the resistance on the remaining frustum may be three fourths of that on the end of a cylinder circumscribing the sphere.

1812.

Monday Morning —Mr. HUSTLER.

MONDAY, JANUARY 15, 1812.

FIRST AND SECOND CLASSES.

1. If two straight lines are parallel, the common section of any two planes passing through them is parallel to either.

2. The length of the tropical year being $365^d\ 5^h\ 48'\ 48''$, explain the reason why three out of four hundredth years are *not* leap-years.

3. Shew the method of discovering whether an equation has any equal roots; and apply it to the solution of the equation,

$$x^5 - 13x^4 + 67x^3 - 171x^2 + 216x - 108 = 0.$$

4. Determine the evolute of the common parabola.

5. Compare the resistance to a given cylinder moving in a fluid in a direction perpendicular to its axis with the resistance to the same cylinder moving with the same velocity in the direction of its axis.

6. At a given hour, on a given day, at a given place, to determine the latitude and longitude of the mid-heaven.

7. If S be the point of suspension of an oscillating body, G the center of gravity, O the center of oscillation; and from center G with radii GS, GO, circles be described in the plane of oscillation; then the axis of suspension being removed to any point in either circumference, the pendulum will oscillate in the same time as before.

8. Find the increment of the number whose hyperbolic logarithm is x.

9. Shew that the fluent of

$$\frac{\dot{z}}{\text{sec.}\ z\,.\ \text{cosec.}\ z} = \frac{1}{4}\ \text{vers. sin.}\ 2z.$$

10. A paraboloid, laid upon a horizontal plane, rests with its axis inclined to the horizon at 30^0. Compare the length of the axis with the latus rectum.

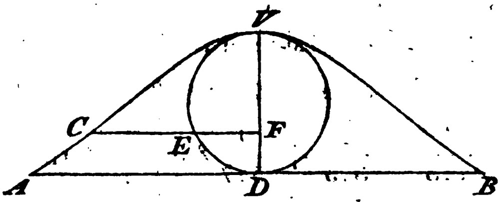
11. If a body be projected at an angle with the velocity acquired in falling from an infinite distance, force varying as $\frac{1}{(\text{dist.})^n}$, compare the chord of curvature of the orbit at the point of projection, with the distance.

12. Sum the following series:

$$\left.\begin{array}{l}\dfrac{5}{1.4} - \dfrac{7}{2.5} + \dfrac{9}{3.6}, \text{ \&c.} \\[2ex] \dfrac{1}{1.3.4} + \dfrac{4}{2.4.5} + \dfrac{7}{3.5.6}, \text{ \&c.}\end{array}\right\} \text{to } n \text{ terms and in inf.}$$

$$1 - \frac{1}{3} + \frac{1.2}{3.4} - \frac{1.2.3}{3.4.5} + \frac{1.2.3.4}{3.4.5.6}, \text{ \&c. to inf.}$$

13. *AVB* is the *trochoid* of Newton, in the sixth Section; *VD* its axis; *CEF* an ordinate parallel to



the base *AB*, cutting the curve in *C*, and the circle on the axis in *E*. Shew that the arc *VE* is to the line *CE* in a constant ratio.

14. If several circles be described, the force tending to a common point in all their circumferences, the periodic times are as the cubes of the radii. Required a proof.

Monday Afternoon.—Mr. Hustler.

FIFTH AND SIXTH CLASSES.

1. Prove that in the multiplication of decimals, there are as many decimal places in the product as in the multiplier and multiplicand together.

2. Find two numbers, such that if $\frac{1}{3}$ of the less be added to $\frac{1}{4}$ of the greater, the sum will be 7; but if $\frac{1}{8}$ of the greater be taken from the less, the remainder will be 2.

3. Extract the square root of $6\sqrt{(-2)}-3$.

4. If a perpendicular be let fall from the vertex of any triangle upon the base, the rectangle by the sides

of the triangle = the rectangle by the perpendicular and the diameter of the circumscribed circle.

5. Having given the sine of an angle, it is required to find the cosine of twice the angle.

6. Sum the series, $2, 2\frac{1}{3}, 2\frac{2}{3}, 3$, &c. to 13 terms.

also, $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7}$ &c. to n terms.

7. Draw a meridian line, and shew the method of finding the sun's meridian altitude by experiment.

8. If a body revolve in a circle, the force being in a point of the circumference; required the law of the force.

9. The square of any odd number increased by 3 is divisible by 4.

10. Shew that the angular velocity in different conic sections about the same focus $\propto \frac{\sqrt{(\text{latus rectum})}}{(\text{dist.})^2}$.

11. Find the fluxion of

$(a+bx^{\frac{3}{2}}+cx^{\frac{2}{3}})^{\frac{1}{2}}$ and of hyp. log. of $\frac{\sqrt{(a^2+x^2)}}{\sqrt{(a^2-x^2)}}$;

also the fluent of $\frac{x\dot{x}+\dot{x}}{x\sqrt{(a^2+x^2)}}$.

12. Prove that all the images of an object placed between two plane mirrors inclined at a given angle will lie in the circumference of a certain circle.

13. Required the amount of £56. 13*s.* 4*d.* put out to simple interest for 5 years and 4 months at 6 per cent. per annum.

14. Prove that the time down different inclined planes $\propto \dfrac{\text{length}}{\sqrt{(\text{height})}}$.

15. If a body float on a fluid, the part immersed : whole body :: specific gravity of body : specific gravity of fluid.

16. Prove that all rays coming parallel to the axis of a parabolic reflector will converge accurately to the focus.

17. In the equation $x^3 - 10x^2 + 27x - 18 = 0$, the greatest root is double of the second and the second treble of the third. Find all the roots.

Monday Afternoon.—Mr. TURTON.

THIRD AND FOURTH CLASSES.

1. Given the series arising from the expansion of $(a+b)^m$; determine the n^{th} term.

2. Find the equation whose roots are $1 \pm \sqrt{(-2)}$, $2 \pm \sqrt{(-3)}$.

3. The roots of the equation $x^3 - px^2 + qx - r = 0$ are a, b, c; find the equation whose roots are ab, ac, bc.

4. Find the sum of n terms of the series $1^2 + 3^2 + 5^2 + \&c.$

5. Find the fluents of the following quantities:

$$\frac{x^5 \dot{x}}{\sqrt{(a^2 - x^2)}}, \quad \frac{\dot{x}}{x^4 \sqrt{(a^2 + x^2)}}.$$

6. Determine, by geometrical construction, that point in a cycloid at which the velocity of an oscillating body is half the greatest velocity.

7. Shew that, at any point in an ellipse, the increase of the focal distance is to that of the perpendicular on the tangent as $CD \times HP$ to $AC \times CB$.

8. A hemisphere, resting on a fluid with its vertex downwards, has two thirds of its axis immersed; compare the specific gravities of the fluid and solid.

9. If a=the altitude of a parabolic frustum, and b, c be the radii of its bases, also p=3.14159; then will the solidity $=\frac{pa}{2} \times (b^2+c^2)$: required a proof.

10. If the reflecting curve be the arc of a given cycloid, the rays being incident parallel to the axis; required the nature and length of the caustic.

11. Determine the points of the compass on which the sun will rise and set, to an observer at a given place, on a given day of the year.

12. Given the earth's radius and the space fallen through in one second at its surface; also the periodic time of the moon; required the moon's distance, gravity varying inversely as the square of the distance from the earth's center.

13. A body descending from rest in a fluid acquires a velocity (a) in falling through the space

(*s*). Compare the specific gravities of the fluid and body, the resistance of the fluid being neglected.

14. Compare the time of moving through the apside, from one extremity of the latus rectum to the other, in different parabolas, round different centers of force in the foci.

Monday Evening.—Mr. Turton.

1. Prove that, if from any point in the directrix of a parabola two tangents be drawn to the curve, those tangents will be at right angles to each other.

2. In treatises on mensuration, the expression $\frac{8 \text{ chord } \frac{1}{2} A - \text{chord } A}{3}$ is given as an approximate value of the arc (A) of a circle: investigate the truth of this approximation.

3. Give Cotes's constructions for determining the orbits described by bodies acted upon by forces varying inversely as the square of the distance.

4. There is a point in the circumference of a circle from which the circumference is suspended. Shew that, if two equal weights be fixed at any points whatever in the circumference, equally distant from the point of suspension, and be made to vibrate in the plane of the circle, the time of oscillation will be equal to that of a pendulum whose

length is the diameter of the circle.—(The circumference is devoid of gravity.)

5. Prove that the time of falling from rest from any point (P) in a parabola to the center of force the focus (S) is to the time of moving in the curve from that point to the vertex (A) as $\frac{3p}{4} \times SP^{\frac{3}{2}}$ to $(SP + 2SA)\sqrt{(SP - SA)}$ ($p = 3.14159$).

6. Solve the fluxional equation $\dot{y} + Py\dot{x} = Q\dot{x}$, where P and Q are functions of x.

7. Shew that, between the values $x = 0$, and $x = 1$,

$$\int \cdot \frac{x^{2n+1}\dot{x}}{\sqrt{(1-x^2)}} = \frac{2.4.6.\ldots\ldots 2n}{3.5.7.\ldots\ldots(2n+1)},$$

and $$\int \cdot \frac{x^{2n}\dot{x}}{\sqrt{(1-x^2)}} = \frac{1.3.5.\ldots\ldots(2n-1)}{2.4.6.\ldots\ldots 2n} \times \frac{\pi}{2}$$

(π = half the circumference of a circle to the radius 1): and from these fluents deduce WALLIS's expression for the circumference of a circle to the same radius; viz. $4 \times \frac{2^2.4^2.6^2.\text{ ad inf.}}{1.3^2.5^2.\text{ ad inf.}}$.

8. If a straight line of given length pass through a fixed point, and one end move along a straight line given in position, construct the curve which will be the locus of the other extremity.

9. Shew in what case the cycloid *within* the globe (in the 10th Section) becomes a straight line: and find, from the requisite data, the time of oscillating in that line.

10. Shew that, when in any curve the velocity is less than that in a circle at the same distance, the angle between the radius vector and *direction of the body's motion* continually diminishes: and that when the velocity is greater than that in a circle the said angle continually increases.

11. Having given two altitudes of a star, whose declination is known, and the times of observation, on a given day, find its right ascension and the latitude of the place.

12. Given the latitude and longitude of three places on the earth's surface; find the latitude and longitude of one equally distant from them all.

13. Given two distances from the pole of a logarithmic spiral, and the angle between them—shew how the spiral may be constructed.

14. Determine the *form* of a vessel of given altitude which being filled with a fluid and a given orifice being opened in the bottom, the velocity of the descending surface will vary as the n^{th} power of the altitude of the surface above the orifice: and find the *content* of the vessel when the surface begins to descend with a given velocity.

15. Shew how to determine, from three observations, the direction in which a comet is moving, supposing the motion to be uniform and rectilinear.

16. If a body be projected from the earth's surface in a direction making an angle of 45^0 with

the horizon, and with the velocity of a body revolving in a circle at the surface; required the point at which it will reach the earth again, and the time of motion.

17. Explain the formation of figurate numbers, and shew that if the figurate numbers of any order be divided by the corresponding numbers of the next order, the sum will be infinite; but that if they be divided by those of the next order but one, the sum will be finite.

18. Determine the conic frustum, of a given base and altitude, on which, when moving in the direction of its axis, with its less end foremost, the resistance will be less than that on any other frustum of the same base and altitude.

19. If a body whirled round by a string, describe a circle in a vertical plane, shew that the string cannot retain the body in the circle, unless it can support six times the weight of the body.

20. Prove that if the center of the generating circle of a cycloid move with half the velocity which a body would acquire in falling through its diameter, the describing point will move in the same manner as a body oscillating in the cycloid.

21. Investigate TAYLOR'S theorem: and shew, from the theorem, that when the ordinate of a curve is either a maximum or minimum, the *first* fluxion vanishes; and that the maximum *or* minimum will be determined by the second fluxion being negative or positive.

22. A triangular prism, with three unequal sides, rests on a fluid with one angle immersed; having given the point in one side through which the surface of the fluid will pass, find the position of the body.

23. A piston, closely fitting a vertical tube, will, by compressing the air as it descends by its own weight, rest at the altitude $\frac{a}{n}$ (a being the whole altitude). Now suppose the piston to be forced down to the altitude $\frac{a}{pn}$ and there left to the action of the compressed air; find the velocity at any point of its ascent; friction being neglected.

24. A prismatic vessel, of given dimensions, with its sides vertical, is filled with a fluid: there are two given and equal orifices, one at the bottom, the other bisecting the altitude; required the time of emptying the upper half, supposing both orifices to be opened at the same instant.

Tuesday Morning.—Mr. Turton.

1. Shew that in addition, subtraction, multiplication and division of quantities of the form $a \pm b\sqrt{(-1)}$, and also in the involution of that quantity, the results will always be of the form $A \pm B\sqrt{(-1)}$.

2. The roots of the recurring equation $x^4+px^2+1=0$ must be of the form a, b, $\frac{1}{a}$, $\frac{1}{b}$: exhibit them in that form.

3. Find the fluent of

$$\frac{d\dot{z}\times(a+bz^n)^{\frac{5}{2}}}{z}, \text{ and of } \frac{d\dot{z}}{z\times(a+bz^n)^{\frac{5}{2}}}.$$

4. Compare geometrically the resistance on the curve of a cycloid, moving in the direction of its axis, with that which would oppose the base.

5. Find, from the requisite data, the actual velocity, acquired in falling through the space AD, in terms of the area $ABFD$. (Newton, Prop. 39.)

6. Give a definition of finite curvature; and determine the nature of the curvature at a point (P) of a curve at which $\frac{QP^3}{QR}$ approaches to a given area (A^2) as its limit.

7. Shew that the expression for the force in the moveable orbit $\left(\frac{F^2}{A^2}+\frac{RG^2-RF^2}{A^3}\right)$, when applied to orbits nearly circular, continually approaches to $A^{\frac{F^2}{G^2}-3}$ as its limit.

8. Having given the latitude and longitude of a star, find its angle of position.

9. Find the annual variation of the right ascen-

sion and declination of a star arising from the precession of the equinoxes.

10. If A be the arc of a circle whose radius is unity, prove that

$$\log. \cos. A = -M\left(\frac{A^2}{2}+\frac{A^4}{3.4}+\frac{A^6}{5.9}+\&c.\right)$$

11. A parabola revolves round its axis, which is vertical, in a given time, and the angular motion will just prevent a body, at any point of the curve, from descending. Required the parameter of the parabola.

12. Prove that, if different reciprocal spirals be described round the same center of force, the areas described in the same time, in those curves, will be equal.

13. Investigate the n^{th} integral of xv.

14. Two weights P and W are connected by a string passing over a fixed pulley; find the velocity of P at any point of its descent from rest, and also the time of descent; the weight of the string and pulley being considered.

Tuesday Afternoon.—Mr. Turton.

FIFTH AND SIXTH CLASSES.

1. Given $x+y=a$, and $x^3+y^3=b^3$; find x and y.

2. Divide 1 by $2a-x$.

3. Find the number from the m^{th} root of which, if the n^{th} root be subtracted, the remainder will be the greatest possible.

4. Shew that the tangent of 45° is increasing twice as fast as the corresponding arc.

5. If a body be kept at rest by three forces, and lines be drawn equally inclined to the directions in which they act, forming a triangle, shew that the sides of this triangle will represent the quantities of the forces.

6. Find the fluent of

$$\frac{bx\dot{x}}{(x-a).(x+a)}, \text{ and of } \frac{x^2\dot{x}}{\sqrt{(a^2+x^2)}}.$$

7. Given the three sides of a plane triangle; shew how the angles may be found.

8. Given the velocity and direction of projection; find the range on a given inclined plane passing through the point of projection.

9. In a given circle inscribe an equilateral triangle; and shew that the square of the side of the triangle is triple the square of the radius of the circle.

10. Given the longest diagonal of a rhombus $=a$, and one angle $=60°$. Required its area.

11. Find the position of a straight line down which the time of falling will be twice the time down the same line when perpendicular to the horizon.

12. Two given weights, A and B, are suspended at the extremities of a uniform straight lever of given length (a) whose weight is W; required the

distance of the fulcrum from one end in case of equilibrium.

13. If the force vary inversely as the cube of the distance, prove that the velocity will vary as the tangent, and the time as the sine of a circular arc, whose radius is the greatest distance, and versed sine the space fallen through.

14. Find how far a body must fall internally, towards the center of force in the focus of an ellipse, to acquire the velocity in the curve, at the point from which it fell.

15. If a body be projected with a given velocity and be acted upon by a given uniformly accelerating force; investigate the principle on which the space described in a given time is determined.

16. Shew that the periodic times in ellipses, described round the same focus, are in the sesquiplicate ratio of their major axes.

17. A given rectangular parallelogram is immersed vertically in a fluid with one side coincident with the surface. Divide it, by a line parallel to the surface, into two parts that will be equally pressed.

Tuesday Afternoon.—Mr. Hustler.

THIRD AND FOURTH CLASSES.

1. Prove that an harmonic mean is less than a geometric.

2. Find the amount of an annuity of £140. payable quarterly, for three years, simple interest being allowed at 5 per cent.

3. In the common parabola, the normal is a mean proportional between the latus rectum and the focal distance. Required a proof.

4. Prove that the chord of 120^0 = tangent of 60^0.

5. A body projected at an angle of 60^0 hits a mark at the distance of 300 feet upon an inclined plane whose elevation is 30^0. Required the velocity of projection, the greatest height above the plane, and the time of flight.

6. Shew that the line joining the moon's cusps is perpendicular to the plane passing through the centers of the Sun, Earth, and Moon.

7. If from the center of an ellipse, with radius equal to the line joining the extremities of the axes, a circle be described; a body let fall from any point in its circumference towards the center, will acquire at the point where it meets the ellipse the velocity which a body revolving in the ellipse about the center would have at the same point.

8. The equations $x^3-3x^2+11x-9=0$, and $x^3-5x^2+11x-7=0$ have a common root; find all the roots.

9. If a cylindrical vessel filled with a fluid and placed perpendicular to the horizon, empty itself

through an orifice at the bottom, shew that the velocity of the descending surface will be uniformly retarded.

10. Prove according to NEWTON'S 39th Prop., that if the force vary as the distance, the velocity in a straight line is as the sine of a circular arc whose radius is the whole distance, and versed sine the space described.

11. Find the fluents of

$$\frac{x^3 \dot{x}}{1+x^2}, \text{ of } x^6 \dot{x} \sqrt{(a^2+x^2)}, \text{ and of } \frac{z^{\frac{1}{2}n-1} \dot{z}}{a^n+z^n}.$$

12. If the angle between the apsides in an orbit nearly circular be 60^0, how does the force vary?

13. If a force varying as the distance tend to the center of a globe, the times of oscillation in all arcs of the hypocycloid are equal.

Tuesday Evening.—Mr. HUSTLER.

1. Two bills, one for £.a payable (b) months after date, the other for £.c payable (d) months after date, are presented at a banker's, who advances £.P for them. Required the rate of simple interest.

2. The excess of the sine above the versed sine is greater for 45^0 than for any other arc less than a quadrant.

3. Find the center of gravity of a portion of a given paraboloid, cut off by any plane.

4. If the arc of a common cycloid in which a pendulum oscillates be divided into four equal arcs, the time through the first quarter = $\frac{1}{3}$ of the whole time of oscillation.

5. In the reciprocal spiral, the tangent of the angle made by the radius vector with the curve varies inversely as the distance. Required a proof.

6. With two dice, compare the chance of throwing the number 7 in one trial with the chance of throwing it twice in three trials.

7. If a body revolve in an ellipse, the force being in one focus, the angular velocity about the other focus is not *accurately* equal to the mean angular velocity, except at four points. Determine those points.

8. In the stereographic projection of the sphere, a great circle not passing through the pole is projected into a circle whose radius is the secant of inclination to the plane of the projection; and the distance of its center from the center of the sphere is the tangent of the same angle.

9. Sum the series

$$\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} \text{ \&c. in inf.}$$

10. The length and distance of a straight object placed before a concave spherical reflector being given, it is required to determine the axes of the image.

11. If several bodies be projected from different distances towards the center, force varying as $\frac{1}{(\text{dist.})^2}$, with the velocities acquired in falling from infinity at those distances respectively; shew, according to Newton's 39th Prop., that the times of falling into the center are in the sesquiplicate ratio of the initial distances.

12. The roots of the equation $x^n - px^{n-1} + qx^{n-2}$ .... $-Qx + R = 0$, being a, b, c, &c. shew that

$$\frac{a^2}{b} + \frac{a^2}{c} + \&c. + \frac{b^2}{a} + \frac{b^2}{c} + \&c. + \frac{c^2}{a} + \frac{c^2}{b} + \&c. = \{p^2 - 2q\} \cdot \frac{Q}{R} - p.$$

13. At a given place, to determine the day when a given star is due south at sun-rise.

14. A sphere acted upon by gravity is projected downwards in a medium with a velocity greater than the greatest acquirable velocity in the medium; determine the velocity after any space has been described, and the limit when the space is infinite.

15. If an ordinate parallel to the base of a common cycloid cut the curve in P and the generating circle in Q, and tangents PT, QT to the cycloid and circle be drawn, meeting in T, the locus of all the points T is the involute of the generating circle.

16. Find the fluents of

$\frac{\dot{x}}{1+x^n}$, (n) being an odd number, of $\frac{\dot{x}\sqrt{(a^2-x^2)}}{x^6}$,

and of $\frac{-\dot{x}.}{\sqrt{\left(\text{hyp. log. } \frac{a}{x}\right)}}$.

17. If the rhumb-line be always inclined to the meridians at 60°, its length from the equator to the pole = half a great circle of the sphere.

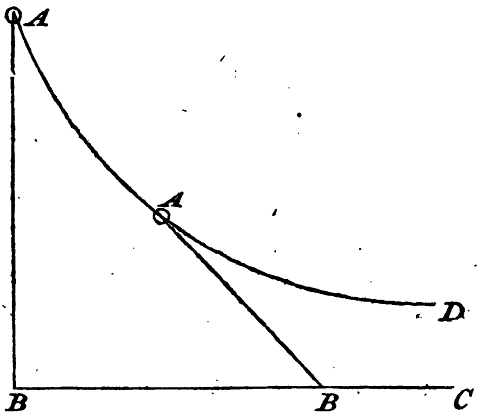
18. Determine the curve of aberration in a plane parallel to the ecliptic, if the orbit of the earth were a circle, and the sun in the circumference.

19. A cube is bisected diagonally by a plane, and one half being filled with a fluid is placed, vertex downwards, with the bisecting plane parallel to the horizon. Find the time of emptying through a small orifice made at the lowest part.

20. Supposing gravity to be constant and perpendicular to the horizon, and that the resistance of a medium varies as density of medium × (vel.)^2 of the body; required the law of density, so that a body may describe a given circle. (NEWTON, Vol. II. Prop. x.)

21. A weight *A* attached to a string *AB*, being laid on a *horizontal* plane *ABCD*, the extremity *B* of the string is moved along a line *BC* which is at first perpendicular to *BA*, and the weight *A* traces out a curve *AD* on the plane. Shew that the

surface of the solid made by the revolution of this



curve indefinite in extent about BC = eight times its area.

22. Find the length of the tide-day at new and full moon, and shew that the tide-day is at all times greater than the solar-day.

23. The first term of an arithmetic series is (a), the last term (l) the common difference (d): also S_1, S_2, S_3 ...... S_{m-1} are the sums of the first, second, third, $(m-1)^{th}$ powers of the terms; shew that $(l+d)^m$

$$-a^m = m.d.S_{m-1} + m.\frac{m-1}{2}d^2.S_{m-2} + m.\frac{m-1}{2}.\frac{m-2}{3}.d^3.S_{m-3} + \&c.$$

24. A great circle revolves about the axis of a sphere with an uniform angular velocity quadruple of that which a point setting off from the pole advances along it. Find the two surfaces into which the motion of the point divides the hemisphere.

1813.

Monday Morning.—Mr. Macfarlan.

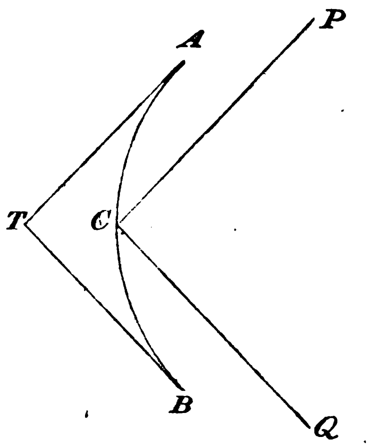
Monday, January 18, 1813.

First and Second Classes.

1. Sum the series, $\cos. A + \cos. 2A + \cos. 3A \ldots + \cos. nA$ where $nA =$ whole circumference. Also the series $\frac{1}{2} - \frac{2}{3} + \frac{3}{4} - \frac{4}{5}$; &c. ad inf.

2. Find the fluent $\frac{\dot{x}}{\sqrt{(1+x^2)} - \sqrt{(1-x^2)}}$; and prove that the fluent of $x^x \dot{x}$ between the values of $x=0$, and $x=1$, is $\frac{1}{1} - \frac{1}{2^2} + \frac{1}{3^3} - \frac{1}{4^4} +$ &c.

3. AB is a spherical reflecting arc; C the middle



point; AT and BT tangents from the extremities, to

which CP and CQ are drawn parallel; it is required in these lines to find two points, P and Q, so that all rays proceeding from P and incident at A or B, may after reflection converge to Q.

4. Find the quantity of refraction by the circumpolar stars, (Boscovich's method) the refraction being supposed to vary as the tangent of the apparent zenith distance.

5. The angle contained by the two equal sides of an isosceles spherical triangle is greater than the angle contained by the chords of the same sides.

6. Find the value of the fraction $\dfrac{a^x - b^x}{x}$ when $x=0$, and of $(1-x)\tan.\dfrac{\pi x}{2}$ when $x=1$, π being $=$ semi-circumference of a circle rad. $= 1$.

7. If a straight line be drawn through the center of gravity of a triangle to meet two sides and the third side produced; the rectangle under the segments of this line measured from the center of gravity on one side of it is equal to the sum of the rectangles under the same two segments and the segment on the other side of the center of gravity.

8. Determine the position in which a lever of given length and uniform thickness will rest between two given inclined planes.

9. If the resistance vary as the velocity, and the force of gravity be constant, the times of describing

all chords of a circle terminating in the extremity of a vertical diameter are equal.

10. In moving from the equator to the pole, the increase of a degree of latitude varies as the square of the sine of latitude.

11. A body revolves in a circle, the center of force being in the periphery. Investigate the nature of the curve traced out by the extremity of the perpendicular upon the tangent; find its area and length, and the value of the greatest ordinate.

12. The density of a lever of given length varies as the n^{th} power of the distance from one extremity, by which it is suspended. A given weight (P) attached to the other end, and acting perpendicularly by means of a pulley, keeps the lever horizontal. The lever (when P is removed) would vibrate m times in t''. Required the weight of the lever, and the index (n).

13. A body, urged by a force varying inversely as the square of the distance, describes from rest a given straight line, while the line itself revolving uniformly performs one complete revolution. Required the area described.

14. The perimeter of an equilateral triangle inscribed in a circle is greater than the perimeter of any other isosceles triangle inscribed in the same circle.

Monday Afternoon.—Mr. MACFARLAN.

FIFTH AND SIXTH CLASSES.

1. Reduce $\dfrac{x^8+x^6y^2+x^2y+y^3}{x^4-y^4}$ to its lowest terms; and prove the Rule for finding the greatest common measure of two quantities a and b.

2. Extract the square root of 315.271, and show generally that if there be n figures in the root there cannot be more than $2n$ nor less than $(2n-1)$ figures in the number whose root is to be extracted.

3. Prove that the hypothenuse of a right-angled triangle is less than the sum of the two sides by the diameter of the inscribed circle.

4. Given the tangents of two arcs; find the tangents of their sum and difference.

5. When the force varies inversely as the square of the dist. the periodic times in ellipses vary as $\dfrac{(\text{axis major})^{\frac{3}{2}}}{\sqrt{(\text{abs. force})}}$.

6. Solve the following equations:

$$\left.\begin{array}{l} xz = y^2 \\ x + y + z = 21 \\ x^2 + y^2 + z^2 = 189 \end{array}\right\} \qquad \sqrt{(5+x)} + \sqrt{x} = \frac{15}{\sqrt{(5+x)}}$$

$$\left.\begin{array}{l} x^2 + y^2 = 34 \\ x^2 - xy = 10 \end{array}\right\}$$

7. Prove that a geometric mean between two quantities is a mean proportional between an arith-

metic and an harmonic mean between the same two quantities, and show of these three mean terms which is the greatest.

8. Extract the square root of $a^3 - b^3$ to four terms by the binomial theorem.

9. Find the fluxions of $(a+x) \times \sqrt{(a-x)}$; of a^x; and the fluent of $\frac{2a\dot{x}}{a^2 - x^2}$.

10. A body is projected from the bottom of a given inclined plane with a given velocity; find the direction when the range will be a maximum.

11. Find the focal length of a glass sphere.

12. The specific gravity of gold and silver being (a) and (b) and of their compound (c). Find the ratio of the quantities of the gold and silver in the mixture.

13. Given the latitude of the place and the sun's altitude at six o'clock; find the time of the year; and give the proportions for solving the spherical triangle.

14. Construct the supplemental triangle, and prove its properties.

15. In the parabola the rectangle under the principal latus rectum and the abscissa is equal to the square of a semi-ordinate to the axis.

16. Prove the rule for the extraction of the square root of a binomial surd, and apply the expression to $7-2\sqrt{3}$.

17. In a triangle whose sides are (a) and (b), and the included angle $\frac{4}{3}$ of a right angle, the square of the base $= \frac{a^3 \sim b^3}{a \sim b}$.

Monday Afternoon.—Mr. Jephson.

THIRD AND FOURTH CLASSES.

1. Investigate the rule for extracting the square root of a binomial surd, and apply it to find the root of $2 + 2\sqrt{(1 - a^2)}$.

2. The true zenith distance of the polar star when it first passes the meridian is 46°. 50′. 40″. 75‴. and at the second passage is 50°. 25′. 50″. 30‴. Required the latitude of the place.

3. If any number of circles be drawn through two given points A and B cutting a given circle, the lines which join the points of intersection shall all meet AB produced in the same point.

4. In a system of (n) equal pullies, each hanging by a separate string, and the strings parallel, having given P and W and the weight of one pulley, find (n) when there is an equilibrium.

5. If a vessel be filled with fluid, the pressure on any part : the weight of the fluid :: area of that part × the depth of its centre of gravity : solid content of the fluid.

6. If $p + 1 : 1$ be the ratio of the tangents of two

angles, and $m : n$ the ratio of their sines, shew that $p+1 : 1$ is always greater than $m : n$.

7. Prove that

$$\int. \frac{\dot{x}}{\sqrt{(x^2+2ax)}} = \tfrac{1}{2} \text{ hyp. log.} \left\{ \frac{\sqrt{x}+\sqrt{(x+2a)}}{\sqrt{2}} \right\},$$

and that

$$\int. \frac{m\dot{z}}{a+bz^2} = \frac{m}{\sqrt{(ab)}} \cdot \text{arc. tang.} = z\sqrt{\frac{b}{a}} \cdot (\text{rad. } 1).$$

8. Sum the series $2^2+4^2+6^2+8^2+$ &c. to (n) terms; and shew that the series $1^3+2^3+3^3+$ &c. to (n) terms, equals the square of the series $1+2+3+$ &c. to (n) terms.

9. Prove the following formulæ,

$$\sin. (x-z) = \text{ultimately } \sin. x - z \cos. x + \frac{z^2}{2} \sin. x,$$

(z) being diminished sine limite;

$$\frac{\cos. A+\sin. A}{\cos. A-\sin. A} = \text{tang. } 2A + \sec. 2A;$$

and having given an arc A, find another arc B so that tang. $B = \sec. A - \text{tang. } A$.

10. Force $\propto \frac{1}{(\text{dist.})^2}$; shew that the velocity at any point of the descent $\propto \tan. \left(\frac{\theta}{2}\right)$; θ being the circular arc whose diameter is the first distance, and versed sine the space described.

11. The increment of a semicircular area made by ordinates perpendicular to the diameter : the contemporary increment of the corresponding sector ::

versed sine of twice the arc : radius. Required proof, and find that point at which the difference between the sector and the area is a maximum.

12. Transform the equation $y^3+2py^2-33p^2y+14p^3=0$ into one whose coefficients are *numeral*.

13. Determine that point in an ellipse, force in focus, where the velocity is a harmonic mean between the greatest and least velocities.

14. Find the content of the greatest cone that can be cut out of a given paraboloid, the vertex of the cone being in the centre of the base of the paraboloid.

15. Find that point in *P*'s orbit (11th section) at which the tangential ablatitious force : the mean addititious :: 3 : 2.

Monday Evening.—Mr. Jephson.

1. Extract the square root of $4mn+2.(m^2-n^2).\sqrt{(-1)}$.

2. There are (p) arithmetical progressions each beginning from unity, the common differences are 1, 2, 3.....p, shew that the sum of their n^{th} terms $=\dfrac{(n-1).p^2+(n+1).p}{2}$.

3. Force to $S \propto \dfrac{1}{(\text{dist.})^2}$; prove that the velocity acquired in descending down any space AC : that which would have been acquired at C if the force

at A had continued constant :: the chord : the sine of a circular arc whose diameter is SA and versed sine AC.

4. If systems of logarithms be taken, whose bases increase in geometrical progression, shew that their moduli decrease in harmonical progression.

5. Given the sun's declination, and that the sun is due east when half the time between his rising and twelve o'clock is elapsed; find the latitude of the place.

6. There are two events A and B independent on each other; the probability of A's happening : probability of failing :: $p : q$; the probability of B's happening : probability of failing :: $r : s$. In $(2n)$ trials what is the probability that they will happen alternately all along?

7. Trace and construct the curve whose equation is $y^2 = \dfrac{x^3 + bx^2}{c - x}$ and determine the acute angle at which it cuts the axis.

8. The concave surface of a cylinder filled with fluid is divided by horizontal sections into (n) annuli in such a manner that the pressure upon each annulus is equal to the pressure upon the base; given the radius of the cylinder, find its height, and also the breadth of the (p^{th}) annulus.

9. Find $\int . \dfrac{\dot{x}}{x . (a^2 + x^2)^{\frac{5}{2}}}$; also having given

$\int . (a+ez^n)^m . z^{vn-1}\dot{z}$, find $\int . (a+cz^n)^{m+p} . z^{vn-1}\dot{z}$; also find the values of y in the equation $y - \frac{\ddot{y}}{\dot{z}^4} = 0$.

10. Sum $\frac{5}{1.2.1.3} + \frac{9}{2.3.3.5} + \frac{13}{3.4.5.7} + \&c.$ to (n) terms and in inf. and

$$\frac{1}{1.3^2} + \frac{2}{3.5^2} + \frac{3}{5.7^2} + \&c. \text{ in inf.}$$

and apply the method of increments to sum (n) terms of the series

$$1 + \frac{3}{1.3} + \frac{3.4}{2.3^2} + \frac{4.5}{2.3^3} + \&c.$$

11. Find the equation to a spiral in which the angle described by the radius vector $SP \propto \frac{1}{SP^n}$; and shew that the subtangent to any point P : the corresponding circular arc described with radius SP and beginning from the asymptote :: n : 1.

12. A revolving spheroid will retain its form if four times the primitive gravity at the equator : five times the centrifugal force of rotation :: semi-axis : the elevation of the equator above the inscribed sphere.

13. Two equal weights sustain each other by means of three tacks situated in the same vertical plane; prove that the *vertical* pressures are together equal to the sum of the weights.

14. An inverted paraboloid is supplied with water at a given rate; given its dimensions, and the area

of the orifice which is in the vertex; it is required to find the highest point to which the water will rise, and also the time.

15. Parallel rays are incident upon a semi-cycloid in a direction perpendicular to the base; find the caustic, and shew that the density $\propto$ tang. θ + 2 tang. 2θ, (θ being that arc of the generating circle of the cycloid which corresponds to the point of incidence.)

16. If (v) = the true anomaly, u = the eccentric, and $e : 1$, the ratio between the eccentricity of the orbit, and its semi-axis major, then

$$\cos. v = \frac{e + \cos. u}{1 + e . \cos. u}.$$ Required proof.

17. A logarithmic curve being described, construct for its sub-tangent.

18. P and Q are placed at the ends of a lever, P hangs by a thread; given P and Q, it is required to find where the fulcrum must be placed so that the tension of the thread may be a maximum.

19. A body oscillating in a medium whose resistance $\propto$ (vel.)^2, construct for the resistance at each point. (NEWT. Prop. xxix. Vol. II.)

20. By means of the compasses alone, it is required to inscribe in a square an equilateral triangle having one angle in an angle of the square.

21. The corner of a rectangular piece of paper is doubled down, so that the triangle shall always be of

a given area, prove that the vertex of the triangle will trace out a lemniscata, whose area equals the area of the triangle and which may be described by a force placed in its *knot* varying as $\frac{1}{D^7}$.

22. A quadrant is stretched out into a straight line, and upon it as an axis ordinates are drawn which are equal to the versed sine of twice the intercepted arc; find the whole area of the curve so traced out, and determine the point of contrary flexure.

23. The nodes being in quadratures, prove that the mean decrement of their motion arising from the acceleration of the areas is equal to $\frac{1}{4}$th of the decrement in syzygy.

24. If a perfectly flexible and uniform chain of a given weight coincide with the convex surface of a vertical quadrant having one radius horizontal, find the velocity acquired in its descent, and the tension at a given point in any given position of the chain.

Tuesday Morning.—Mr. Jephson.

FIRST AND SECOND CLASSES.

1. Are the quantities $\sqrt[4]{(-a)} \times \sqrt[4]{(-b)}$ and $\sqrt[6]{(-a)} \times \sqrt[6]{(-b)}$ possible or impossible?

2. Given that a parabolic area : its circumscribing parallelogram always :: m : n; it is required to find the ratio between the solids generated by these

areas revolving round their common axis, and to apply it to the case of the common parabola.

3. Shew that at a point of contrary flexure the fluxion of $\frac{\dot{x}}{\dot{y}} = 0$.

4. The inscribed sphere is taken away from an oblate spheroid of small eccentricity; find that annulus parallel to the plane of the equator which attracts a corpuscle at the pole with a maximum force.

5. If $(-a)$ be a root of the equation

$$x^n + px^{n-1} \mp qx^{n-2} \pm, \&c. \ldots\ldots \pm W = 0,$$

shew that $x+a$ is a divisor of the *expression*

$$x^n + px^{n-1} \mp qx^{n-2} \pm, \&c. \ldots\ldots \pm W.$$

6. The latitudes of two places on the same meridian are observed, and perpendiculars to their horizons are drawn meeting within the earth, (not supposed to be a sphere); shew that the angle at which the perpendiculars will meet is equal to the difference of the observed latitudes.

7. Find the content of the solid generated by the revolution of a cycloid round a tangent parallel to the base.

8. Explain the Nonius, and shew that the instrument is rendered more *sensible* by increasing the number of the divisions.

9. A bag contains three red balls and two white ones; what is the probability of taking out a white ball in two trials?

10. Give a geometrical construction for finding the resistance upon any curve, and apply it to determine the ratio between the resistance upon a catenary moving in the direction of its greatest ordinate and the resistance upon its axis.

11. The circumference of a semi-circle being considered as an abscissa, and ordinates drawn from its convex side in a direction perpendicular to the diameter and varying as the $n^{\text{th.}}$ power of the intercepted arc, it is required to draw a tangent to the quadratrix thus traced out, and to shew that in the case of the cycloid, the tangent is parallel to the corresponding chord of the generating circle.

12. A perfectly hard body falls down an inclined plane AB, and is inflected along another inclined plane BC; now AD, DE, EF are respectively perpendicular to CB produced, to AB and to the horizontal line CF; shew that the velocity acquired at C is equal to that which would be acquired by falling down EF.

13. Every recurring series whose scale of relation is $f-g+e$, may be resolved into three geometrical series, whose common ratios are the roots of the equation $x^3-fx^2+g\dot{x}-e=0$.

14. P hanging freely raises W up an inclined plane by means of a thread not parallel to the plane; find the tension of the string.

15. If (t) = the time of a comet's passage between

its nodes, $(q)=$ one year, $\pi=3.14159$, then will the line of nodes $=\sqrt[3]{\frac{(18\pi^2t^2)}{q^2}}$, the earth's mean distance being (1). Required proof.

Tuesday Afternoon.—Mr. JEPHSON.

FIFTH AND SIXTH CLASSES.

1. Shew that sin. $(-A)=-(\sin. A)$, and that cos. $(-A)=\cos. A$. Is sec. $(-A)=+$ or $-$ sec. A?

2. Solve the following equations:

(1.) $a+x+\sqrt{(2ax+x^2)}=b$ }

(2.) $$\left.\begin{array}{l} \frac{y}{x}-\frac{x}{x+y}=\frac{x^2-y^2}{y} \\ \frac{x}{y}-\frac{x+y}{x}=\frac{y}{x} \end{array}\right\}$$

3. The earth a sphere, and its radius 4000 miles, find what distance may be seen by a person 9 feet high.

4. Divide x^3-px^2+qx-r by $x-a$.

5. If two equal forces sustain each other by means of a string passing over a tack, shew that either of the forces : pressure upon the tack :: radius : 2 cosine of half the angle at which the forces act.

6. If $a : b :: c : d$, then will $a \pm mb : b :: c \pm md : d$.

7. Prove that the radius of a circle which bisects the chord will bisect the arc.

8. Shew that the versed sines of the same angle in different circles are to each other as the radii of the circles.

9. If $A : B :: C : D$, then will log. $D =$ log. $B +$ log. $C -$ log. A.

10. An inverted paraboloid is filled with a fluid; find that horizontal section which sustains the greatest pressure.

11. Explain the magic lantern.

12. Shew the use of logarithms in finding the value of $\frac{A^2\sqrt{(B^2-C^2)}}{C\sqrt[5]{(D^3.E.F)}}$.

13. Find all the combinations which can be made out of the letters of the word *Baccalaureus*.

14. A certain velocity (a) is communicated to each of two perfectly hard bodies at the instant of their impinging on each other. Shew that the common velocity after impact equal $a \pm$ what would have been the common velocity if (a) had not been communicated.

15. Find the 28th term of the series 13, $12\frac{2}{3}$, $12\frac{1}{3}$, &c. and sum the series $2, -\frac{1}{3} + \frac{1}{18}, -\frac{1}{108}, +$ &c. in inf.

16. Take the fluxion of $(x^m + bx^n)^p$ and find the

$$f.\ \frac{\dot{x}}{a-mx}, \text{ and } f.\ \frac{z^{\frac{n}{2}-1}\,\dot{z}}{a^n+z^n}.$$

17. Prove that in all curves the centripetal force : the centrifugal :: $SP : \frac{1}{2} \cdot \frac{QT^2}{QR}$.

18. Force to S varying as dist.; shew that the velocity acquired in descending through any space AC : that which would have been acquired at C if the force at A had continued constant :: sine : the chord of a circular arc whose radius is SA, and versed sine AC.

Tuesday Afternoon.—Mr. MACFARLAN.

THIRD AND FOURTH CLASSES.

1. Solve the equation $x^3 - 11x^2 + 36x - 36 = 0$, the roots being in harmonical progression.

2. If (n), a prime number, be the index of a binomial, every term of the expanded binomial, except the first and last, is divisible by (n).

3. If two bodies acted upon by constant moving forces in the proportion of 5 : 4, describe spaces from rest in the proportion of 4 : 5, and acquire velocities in the proportion of 5 : 6. Required the ratio of the quantities of matter.

4. In a given circle, the plane of which is vertical, to draw a diameter, which shall be described by a heavy body in any given time, not less than the time in which the vertical diameter is described.

5. The least angle which can be made with the horizon, by any great circle passing through the

place of a star at any given time, is measured by the star's altitude.

6. The periodic times in all ellipses round the same center are equal.

7. Find the effects of precession in right ascension and declination; and shew when the effect in right ascension vanishes.

8. Find the fluent of

$$\frac{x^3\dot{x}}{\sqrt{(2ax-x^2)}}, \text{ and of } \dot{x}\times(\sin. x)^2.$$

9. Sum the following series:

(1.) $1^2 + 3^2 + 5^2 + 7^2$ to 12 terms.

(2.) $1.x + 2x^2 + 3x^3 + 4x^4 \ldots + nx^n$.

(3.) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2}$, &c. ad inf.

10. The density of the sun's rays formed by a spherical reflector : density of his rays formed by a glass sphere of the same radius :: 9 : 1. Required proof.

11. Compare, after Newton's method, the resistance upon the solid generated by the revolution of a cycloid round the base, moving in the direction of the base, with the resistance upon the circumscribing cylinder.

12. Having given the angle of a plane triangle, the side opposite to it and the sum of the other two sides; required the sides.

13. Let a sphere descend by its gravity in a fluid, whose specific gravity is to that of the sphere as 1 to n. Find the greatest velocity it can acquire on supposition that the resistance varies as the square of the velocity.

14. Extract the square root of 2 by a continued fraction.

Tuesday Evening.—Mr. Macfarlan.

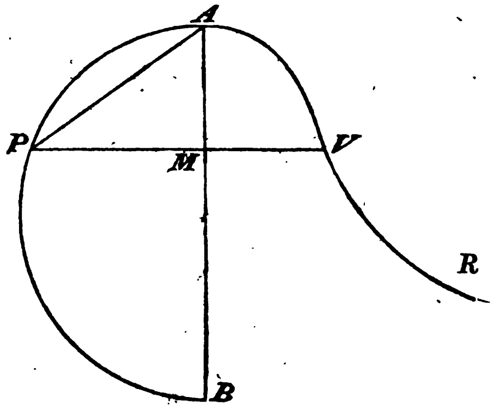
1. If the national debt be A£. and P£. be annually invested at compound interest as a sinking fund, in how many years will the debt be discharged, the interest of A not being considered?

2. Find the sun's place in the ecliptic, when the aberration of a given star in declination vanishes; and shew that the aberration in right ascension is not necessarily a maximum when the aberration in declination $=0$.

3. Find, after Newton's manner, the law of force whereby a body may describe a semi-ellipse, the direction of the force being perpendicular to the axis major.

4. When the force varies inversely as the fourth power of the distance, and a body is projected from an apse with the velocity acquired in falling from an infinite distance, to define the orbit.

5. The curve AVR and the semi-circle APB have the same abscissa; the ordinate MV is equal



to the tangent of half the arc AP. Prove that the area AMV is equal to twice the circular segment AP; and find the point of contrary flexure.

6. Sum the series

$$\frac{5}{1.2.3}+\frac{6}{2.3.4}+\frac{7}{3.4.5} \text{ to } n \text{ terms,}$$

and $$\frac{6^2}{1.2.3.4}+\frac{7^2}{2.3.4.5}+\frac{8^2}{3.4.5.6} \text{ to } n \text{ terms}$$

and ad infinitum.

7. Find the chance of throwing three aces exactly in five throws with a single die.

8. Find the greatest of all triangles having the same vertical angle and equal distances between that angle and the bisection of the opposite side.

9. In orbits of little eccentricity, the greatest equation of the center is equal to twice the eccentricity.

10. Integrate the following fluxional equations:

$$\frac{\dot{y}}{y} - \frac{\dot{x}}{x} = \frac{x^m \dot{x}}{ay\sqrt{n}};$$

$$x\dot{x} + a\dot{y} = b\sqrt{(\dot{x}^2 + \dot{y}^2)}.$$

$$y\dot{x} - x\dot{y} = \dot{x}\sqrt{(x^2+y^2)}.$$

11. Find the fluent of

$$\frac{\{1+\sqrt{(-x)}\sqrt[3]{x^2}\}\dot{x}}{1+\sqrt[3]{x}}, \text{ and of } \frac{\dot{x}}{(1+x^2)^n};$$

and when $n=3$ shew that the value of the latter fluent, between the value of $x=0$ and $x=1$, is $\frac{1}{4}+\frac{3A}{8}$, A being an arc of 45° to radius 1.

12. If any number of projectiles be thrown from the same elevated point, and with the same velocities in an horizontal direction; the locus of the points in which they will strike a given inclined plane will be a conical frustum.

13. If a cylindrical vessel, placed vertically, and kept full of water, be bored in innumerable points; the issuing fluid will be bounded by the surface of a conical frustum.

14. Find the equation between the abscissa and ordinate of the catenary.

15. A body (P) draws a lighter body (W) over a fixed pulley. A small oscillating motion is given to W at the commencement of P's action. Find the number of oscillations before W reaches the pulley: show that this number is independent of the string's length; and that however great P is, $\frac{1}{\sqrt{2}} \times$ oscillation at least will be performed in the time specified.

16. Two rods, the one 6, the other 8 feet high, are placed on a given day perpendicularly to the horizon, at the distance of 20 feet from each other. During the forenoon the extremity of the shadow of the first rod falls at the base of the second. In the afternoon the extremity of the shadow of the second falls at the base of the first. Required the latitude of the place, and the azimuth of one rod seen from the other.

17. If a body be projected from an apse with a given velocity, the force acting perpendicularly to a given plane, and varying in some inverse ratio of the distance from it, investigate the fluxional equation to the curve which will be described, and apply it to the case where the force is constant.

18. Supposing the sun to move uniformly on in a right line with a given velocity, and the earth revolving round him, to preserve always the same distance, it is required to define the earth's path in fixed space.

19. A body describes the quadrant of a circle touching a vertical line at its highest point, being urged by a force perpendicular to the horizon. Required the law of force which will make it recede uniformly from the horizontal radius, and the time elapsed and the velocity acquired at any point of the descent.

20. If any number, a multiple of 11, and a number consisting of the same digits in an inverted order, be each divided by 11, the sum of the digits in the two quotients are equal. Required a proof.

21. If n be a prime number, the product of $1 \times 2 \times 3 \times 4 \dots \times (n-1)$ when increased by unity is divisible by n.

22. If the coefficients of each term of the expanded binomial $(a-b)^n$ taken in order, be multiplied by the terms of the progression 1^m, 2^m, 3^m, &c. taken in order, the result is equal to nothing, n and m being integers, and n greater than m. Required proof.

23. Let α, β, γ, &c. be the roots of the equation

$$x^n - nax^{n-2} + n.\frac{(n-3)}{2}a^2x^{n-4} - \&c. = -b; \text{ then}$$

$$\alpha^{2r} + \beta^{2r} + \gamma^{2r} + \&c. = \frac{2r.(2r-1).(2r-2)\dots(r+1)}{1.2.3\dots\dots\dots r}$$

$\times na^r$ ($2r$ less than n.) Required a proof.

24. A body, whose weight is W falls down the length of an inclined plane, which has the power of

moving freely along an horizontal plane, on which it stands. Given the weight of the prismatic figure composing the plane, it is required to find the path of the body W, the time of describing it, and the last acquired velocity of the moveable plane along the horizontal plane.

1814.

Monday Morning —Mr. Bland.

Monday, January 17, 1814.

FIRST AND SECOND CLASSES.

1. The time of a body's falling through half radius by the uniform action of the centripetal force in the circumference of a circle is to the periodic time as radius is to the circumference of the circle. Required a proof.

2. Prove, without resolving the equation into factors, that if two numbers, a and b, when substituted for the unknown quantity in the equation $x^n - px^{n-1} + qx^{n-2} - \&c. = 0$, give results affected with contrary signs, there is at least one root between a and b.

3. If a line intercepted between the extremity of the base of an isosceles triangle, and the opposite side (produced if necessary) be equal to a side of the triangle, the angle formed by this line and the base produced is equal to three times either of the equal angles of the triangle.

4. Given the greatest range of a projectile upon an horizontal plane; determine at what distance from

the point of projection an object, whose perpendicular height is (d), must be situated, so that the projectile may just strike the top of it.

5. From the vertex of a paraboloid of given dimensions, a part equal to one-fourth of the whole is cut off by a plane parallel to the base; and the frustum being then placed in a fluid with its smaller end downwards, sinks till the surface of the fluid bisects the axis which is vertical. It is required to determine the specific gravity of the paraboloid, that of the fluid in which it is immersed and the density of the atmosphere being given.

6. Given the fluent of $\dfrac{d\dot{z}}{z\,.\,(a+cz^2)^2}=A$,

to find the fluent of $\dfrac{d\dot{z}}{z^3.(a+cz^2)^3}$;

and find the algebraical relation of x to y, in the equation $y^2\dot{y}=3yx\dot{x}-x^2\dot{y}$.

7. Find the sum of the series

$$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\text{\&c. in inf.}$$

and $\dfrac{10.18}{2.4.9.12}+\dfrac{12.21}{4.6.12.15}+\dfrac{14.24}{6.8.15.18}+$ &c. to n terms.

8. Require the nature of the curve such, that if any point P in it be taken, and an ordinate PN and normal PG be drawn to the axis; then if the triangle PNG be placed in such a position that the sub-normal NG may become the ordinate, PG will be the normal.

9. To spectators situated within the tropics, the sun's azimuth will admit of a maximum twice every day, from the time of his leaving the solstice till his declination becomes equal to the latitude of the place. Required a proof.

10. If parallel rays fall upon a single thin lens of given substance; determine the diameter of the least circle into which all the rays of different colours are collected, the linear aperture of the lens being known.

11. Compare the magnitude of that part of the disturbing force of the sun on the moon, which acts perpendicularly to the plane of the moon's orbit, with the moon's gravity to the earth.

12. The velocities of two bodies A and B are in a given ratio, and they begin to move at the same time from A and B, the extremities of a given line AB; A moving uniformly in a direction inclined at a given angle to AB, and B uniformly in the direction BA. Determine the nature of the curve, to which the line joining the bodies is always a tangent.

13. The moon's nodes complete a revolution in about 19 years. Determine the periodic time of the nodes of the third satellite of Jupiter, which revolves in about seven days, Jupiter's period being about 12 years.

14. If (a) be the number of chances for the happening of an event, and (b) the number for its

failure in each single trial; find the probability of its happening p times and failing q times in $(p+q)$ trials; and determine how many trials are necessary to make it an even chance whether the event will happen or not.

Monday Afternoon.—Mr. Bland.

FIFTH AND SIXTH CLASSES.

1. Extract the square root of $\frac{9.0240160}{25.3009}$.

2. Prove the rule for completing the square in a quadratic equation.

3. Find the sum of the series

$$-9-7-4-\&c. \text{ to } 20 \text{ terms,}$$

$$\text{and } 1+\frac{2}{3}+\frac{4}{9}+\&c. \text{ to } 10 \text{ terms,}$$

$$\text{and } 1.2+2.3+3.4+\&c. \text{ to } n \text{ terms.}$$

4. If three quantities are in an increasing arithmetical progression; shew that the second will have to the first a greater ratio than the third to the second.

5. The weights of two perfectly elastic balls are 11 and 8, and their velocities in opposite directions are 12 and 7. Required their velocities after impact.

6. Let the height of an inclined plane be (a) feet, and its length (c) feet. Find the time of a body's descending down (a) feet of the plane.

7. Two fluids, whose magnitudes and specific gravities are given, being mixed together; the magnitude of the mixture : sum of the magnitudes of the ingredients :: n : 1. Determine the specific gravity of the mixture.

8. Compare the time in which any prismatic vessel is emptied by an orifice in the lower surface, with the time of a heavy body's falling through a space equal to twice the depth of the orifice.

9. Divide a right line into two parts, such, that their rectangle may be equal to a given square; and determine the greatest square that the rectangle can equal.

10. Find the fluxions of

$$(a+cz^n)^m \times zp, \text{ and of } x.\frac{\sqrt{(1+x^2)}}{\sqrt{(1-x^2)}};$$

and find the fluents of

$$\frac{\dot{x}}{x\sqrt{(x^2-a^2)}}, \quad \frac{x^3\dot{x}}{a^4-x^4}, \quad \text{and } x^2\dot{x}\sqrt{(a^2+x^2)}.$$

11. Construct Newton's Telescope, and find its magnifying power.

12. Explain the reason why the order of the colours is inverted in the secondary rainbow.

13. Given the sun's altitude at six o'clock, and also when due east; find the latitude of the place.

14. If from a quantity which varies as $\frac{1}{A^2}$, any

quantity be subtracted which varies as A, the remainder will vary in a higher inverse ratio than the inverse square of A; but if to a quantity varying as $\frac{1}{A^2}$ another be added which varies as A, the sum will vary in a lower inverse ratio than the inverse square of A. Required proof.

15. Find the law of the force tending to the centre of the logarithmic spiral.

16. Prove that when the force acts in parallel lines, the velocity in the direction perpendicular to the direction in which the force acts, is constant.

17. If the altitude of a cylinder be equal to the diameter of its base, the whole surface is six times the area of the base.

18. If $a^{mx}\,b^{nz}$ be constant, and $(mx+n).(nz+m)$ be a maximum; prove that $a^{mx+n}=b^{nz+m}$.

Monday Afternoon.—Mr. MACFARLAN.

THIRD AND FOURTH CLASSES.

1. Required the perpendicular from the vertex upon the base of a triangular pyramid, all the sides of which are equilateral triangles of a given area.

2. Given the difference of the times of setting of two stars whose declinations are known; find the latitude of the place.

3. Find the center of oscillation of a conical surface suspended by the vertex; and find the ratio

between the radius of the base and the axis, when the center of oscillation is in the base.

4. The length of a pendulum oscillating seconds on the earth's surface being given; find the length of a pendulum oscillating seconds at the distance of the earth's radius from the surface. Then determine a point below the surface where the last pendulum will vibrate in the same time.

5. Two roots of the equation $x^4+x^3-11x^2+9x+18=0$ are of the form $+a$, $-a$. Find all the roots.

6. When the force varies as that power of the distance whose index is $(n-1)$. Shew that the velocity of a body falling from rest varies as $\sqrt{\frac{(P^n-A^n)}{n}}$, where P is the greatest and A the variable distance. And find the value of this expression, when the force varies inversely as the distance.

7. If from the extremity of the diameter of a circle tangents be drawn and produced to intersect a tangent to any point in the circumference, the right lines joining the points of intersection and the center of the circle shall form a right angle.

8. Sum the series

$$\frac{1}{1.3}-\frac{1}{2.4}+\frac{1}{3.5}-\frac{1}{4.6}+\text{ \&c. to } n \text{ terms,}$$

and $\frac{1}{2.3}+\frac{1}{4.5}+\frac{1}{6.7}+\frac{1}{8.9}$ ad inf.

9. Find the fluents of

$$\frac{\dot{x}}{\sin. z \times \cos. z}, \text{ and } \frac{\dot{x}}{\sqrt{(A+Bx-Cx^2)}}.$$

10. Find the attraction of a sphere on a particle of matter placed at any distance from the center, the force of each particle varying inversely as the cube of the distance.

11. Find the equation to the curve, the length of whose tangent between any point and the axis is a constant quantity.

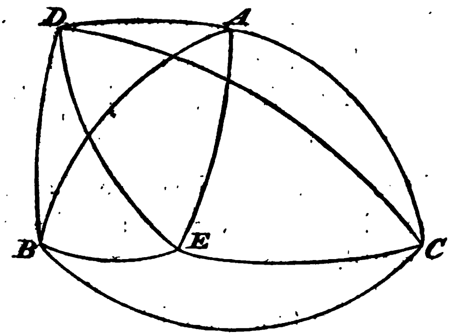
12. The equation to a curve is $y^3 - axy + x^3 = 0$. Find the value of the ordinate when a maximum, and the corresponding value of the abscissa; and show that the above is a maximum and not a minimum.

13. A paraboloid placed with its vertex downwards being full of water, is supplied at a given rate. There is a small hole in the vertex, which, when the vessel is full, would discharge n times the quantity supplied. Required the altitude at which the surface remains stationary, and the time elapsed before this takes place.

Monday Evening.—Mr. Macfarlan.

1. A body placed in the center of gravity of a triangle is acted upon by three forces represented in quantity and direction by the lines joining the center of gravity with the three angles. Show that the body will remain at rest.

2. The sides of a spherical triangle ABC are each a quadrant. D and E (any two points on the surface of the sphere) are joined by the arc of a



great circle. Show that the cosine of DE is equal to the cos. $AD \times$ cos. AE + cos. $BD \times$ cos. BE + cos. $CD \times$ cos. CE.

3. If the sum of the odd digits in a number be $11m+e$ and of the even $11n+e$, this number being divided successively by 11 and by 9, leaves the same remainder as $m+n+e$ when divided by 9.

4. In a dial for a given latitude, the plate of which ought to have been horizontal, the interval between ten and noon is less by two minutes, than the interval between noon and two o'clock. The line between north and south was found to be horizontal. Required the dip of the plate towards the east.

5. A sphere filled with water empties itself through a small hole in the bottom; find where the velocity of the surface of the descending fluid is a

minimum, and where it is equal to the velocity when the sphere is half full.

6. The mean apparent diameter of the sun and moon's horizontal parallax being given, together with the length of a year and a month, find the density of the sun compared with the density of the earth; also shew how Newton finds the density of the moon.

7. From Newton's construction for the solid of least resistance, shew that in the section through the axis, the curve must make with the end an angle of 135°.

8. If the quiescent orbit be a circle (the center of force in the circumference) and the angular velocity in the moveable orbit is double that in the quiescent; Find the law of force in the orbit in fixed space, and investigate the ratio between the perpendicular and distance.

9. A wooden ball connected by a small wire with a ball of lead of the same diameter is dropt into the sea, and upon their striking the bottom, the wooden ball is disengaged and rises to the surface; the whole time elapsed, and the specific gravities and diameters of the balls being given; find the depth of the sea.

10. In any conic section, if tangents be drawn at the extremities of any diameter, and be produced to meet a tangent to any other point in the curve; prove that the rectangle under the segments of the

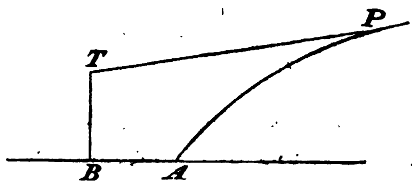
first tangents will be equal to the square of the semi-conjugate diameter.

11. $n^n - n(n-2)^n + n.\frac{n-1}{2}.(n-4)^n - n.\frac{n-1}{2}.\frac{n-2}{3}.(n-6)^n$ &c. continued to $\frac{n}{2}$, or $\frac{n+1}{2}$ terms, is equal to $1 \times 2 \times 3 \times \ldots\ldots n \times 2^{n-1}$. The demonstration is required.

12. A given number (n) of similar balls being put into an urn; required the chance of drawing an odd, to the chance of drawing an even number; any number, from 1 to n inclusive, being equally likely to be drawn.

13. Find the equation and construct the curve of which this is the property: if from a fixed point in the axis a perpendicular be drawn to it, and produced to meet a tangent to any point in the curve, the length of this perpendicular and tangent together shall be double the length of the curve between the vertex and the point from which the tangent was drawn.

In the figure annexed, $BT + TP = 2AP$.



Does the curve admit of an asymptote?

14. In a medium resisting as the square of the velocity, show that a perfectly elastic body falling from an infinite height will at each rebound rise through spaces proportional to the logarithms of the fractions $\frac{2}{1}, \frac{3}{2}, \frac{4}{3} \ldots\ldots\ldots\ldots \frac{n+1}{n}$.

15. Sum the series

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \text{\&c. ad inf.}$$

and $1^3 + 3^3 + 5^3 + 7^3$ to n terms, by increments.

16. Given the fluent of $(e + fx^n)^m \times x^p \dot{x}$,

find the fluents of $(e + fx^n)^m \times x^{p+n} \dot{x}$,

and of $(e + fx^n)^{m+1} \times x^p \dot{x}$;

also find the fluent of $\dfrac{a^{\frac{1}{2}} + y^{\frac{1}{2}}}{y^{\frac{1}{2}} + y^{\frac{2}{3}}} \times \dot{y}$.

17. Solve the following fluxional equations,

$$x^2\dot{y} + 2xy\dot{y} = b^2\dot{x} - y^2\dot{y}$$
$$cx^2\dot{x} + y\dot{x} = a\dot{y}.$$

18. Show that the log. $(1 + n.\cos. z)$ is equal to $A + \frac{2B}{1}\cos. z + \frac{2B^2}{2}\cos. 2z + \frac{2B^3}{3}\cos. 3z + \frac{2B^4}{4} \times \cos. 4z$ &c. where A is the log. of $2B$, and $B = \frac{1-\sqrt{(1-n^2)}}{n}$.

19. In a medium resisting as the square of the velocity, find the nature of the curve to be described by a heavy body urged by the force of gravity, so that the times of descent through different arcs

to the same fixed point shall vary as the velocity acquired.

20. Four persons (A, B, C, D) to whom the cards of a common pack are dealt in order, one by one, stake each £1. with the condition, that he to whom the first knave is dealt, shall be the winner. What is the value of A's expectation?

21. Find the curve by the revolution of which round an axis the solid will be formed, which shall attract a particle placed at its vertex with the greatest possible force, the force of each particle varying inversely as the square of the distance.

22. A cylindrical vessel full of water is balanced by a weight P over a fixed pulley. A hole of given dimensions being made in the bottom, it is required to find how far P will descend during the time of emptying.

23. Prove that the sum of the reciprocals of the prime numbers is an infinitely great number, though infinitely less than the sum of the reciprocals of the natural numbers.

Tuesday Morning.—Mr. Macfarlan.

FIRST AND SECOND CLASSES.

1. A person borrowed P£. at interest. To discharge this he invested £2. at the end of the first year, £4. at the end of the second, and 8£. at the

end of the third, and so on. How many years will elapse before this fund be large enough to discharge the debt,—compound interest being allowed on both sides at a given rate?

2. Required the length of a spherical shell of iron, which, when filled with a fluid, shall just float in water; the specific gravities of iron, of water, and of the fluid, being given.

3. Compare the length of a degree of latitude at any place on the earth's surface, with the length of a degree of longitude at the equator.

4. The inclination of a small tube in the side of a vessel of water being given, and its height above the horizontal plane; it is required, from observing the point of the plane struck by the stream, to assign the altitude of the water within the vessel; and to describe the whole track of the issuing fluid.

5. If round any point within the circumference of a circle (not being the center) equal adjoining angles be described; of the circumferences on which they stand, that which is nearer the diameter passing through the point is less than the more remote.

6. In a combination of two wheels and axles, the circumference of each wheel is n inches; of each axle, 1. A weight, P, is applied to the circumference of one of the wheels as a power to raise matter to a certain height. How much must be raised each time, that the whole quantity may be raised in the least time possible?

7. Of all cones containing a given quantity of matter, to find that which attracts a particle placed at its vertex with the greatest possible force.

8. Show that when a quantity is a maximum or a minimum, the first fluxion vanishes; and that the quantity is a maximum or a minimum, accordingly as the second fluxion is negative or positive.

9. An imperfectly elastic ball is projected with a given velocity against an hard horizontal plane, and being reflected, just reaches the point of projection in t''. Required the distance of the plane, from the point of projection, and the elasticity of the body.

10. A cylindrical tube of given length, closed at one end, being let down in a vertical position into the sea, it was observed what part of the tube the water occupied. It is hence required to assign the depth, 33 feet of sea water being assumed to measure the weight of the atmosphere. How must this tube be graduated to be used as a gage to measure depths in the sea?

11. Find the length of a common parabola, and deduce Cotes's construction.

12. If $x = \dfrac{\epsilon^{az} - \epsilon^{bz}}{2a}$, where a and b are roots of the equation, $v^2 + 1 = 0$:

Show that $\dot{z} = \dfrac{\dot{x}}{\sqrt{(1-x^2)}}$.

13. Sum the series

$$1^2+3^2+7^2+15^2 \text{ \&c. to } n \text{ terms},$$

and $\frac{1}{2.3}\times\frac{1}{2}+\frac{1}{3.4}\times\frac{1}{2^2}+\frac{1}{4.5}\times\frac{1}{2^3}+$ &c. ad inf.

Tuesday Afternoon.—Mr. MACFARLAN.

FIFTH AND SIXTH CLASSES.

1. Find the value of £.12341414141 &c.

2. The amount of £500. in $\frac{3}{4}$ of a year was £520. Required the rate per cent.

3. Find the circumference of a circle whose radius is unity.

4. Sum the following series,

(1.) $2+3+\frac{9}{2}+$ &c. to 20 terms,

(2.) $\frac{1}{1.3}+\frac{1}{2.4}+\frac{1}{3.5}+$ &c. to n terms,

and $\frac{1}{1.2.3}+\frac{1}{3.4.5}+$ &c. ad inf. by increments.

5. Solve the following equation whose roots are in arithmetic progression;

$$x^3-9x^2+23x-16=0.$$

6. Find the fluents of $\frac{a\dot{x}}{a^2-x^2}$, $\frac{x^2\dot{x}}{\sqrt{(a^2+x^2)}}$,

and the fluxion of $\frac{(x+a)^2}{\sqrt{(x^2-a^2)}}$.

7. The arc of a circle which a body, acted upon by a centripetal force, uniformly describes in any given time is a mean proportional between the diameter of the circle, and the space described by a heavy body from rest in the same time when urged by the force in the circumference continued uniform.

8. Show that the logarithm of $(1+u)$ is equal to

$$u - \frac{u^2}{2} + \frac{u^3}{3} - \frac{u^4}{4} + \frac{u^5}{5} - \&c.$$

9. Given the radii of the surfaces of a double convex lens and the ratio of the sines of incidence and refraction. Find the focal length.

10. Find the latitude of the place at which the sun sets at three o'clock on the shortest day.

11. When the force varies as the distance, the periodic time in all ellipses round the same center are equal.

12. A body (A) weighs 12lbs. in vacuo, and 9lbs. in water; another body (B) weighs 10lbs. in vacuo, and 8lbs. in water. Compare their specific gravities.

13. If the number of mean proportionals interposed between two elastic bodies A and X be increased without limit, the velocity of A will be to the velocity communicated to X by means of the intermediate bodies :: $\sqrt{X} : \sqrt{A}$.

14. The apparent diameter and declination of the sun being given, find the time of his transit over the meridian.

15. The plane of a circle being vertical, and any number of chords being drawn to the lower extremity of the vertical diameter; find the locus of any number of heavy bodies falling together from the upper extremities of the diameter and the chords at any given instant of time.

16. If any number of projectiles be thrown at the same instant from the same point and with equal velocities, but in several directions in the same vertical plane, they will at the expiration of any time all be found in the circumference of some circle.

Tuesday Afternoon.—Mr. Bland.

THIRD AND FOURTH CLASSES.

1. Shew from the principles of the fifth book of Euclid, that a ratio of greater inequality is diminished, and of less inequality increased, by adding a quantity to both its terms.

2. The time of day at a given place determined from observations of the sun's altitude is $9^h. 10'. 45''$; and a chronometer set to Greenwich time shews $6^h. 3'. 10''$. Required the longitude of the place of observation from Greenwich.

3. In any harmonic progression, the product of the two first terms is to the product of any two adjacent terms as the difference between the two first is to the difference between the two others. Required a proof.

4. An object is placed between two plane reflectors, which are inclined to each other at an angle of 60°. Determine the whole number of images formed by the reflectors.

5. If the greatest possible rectangle be inscribed in the quadrant of a given ellipse, shew that the elliptic areas cut off by the sides of the rectangle are equal.

6. Prove that an equation of an odd number of dimensions, and an equation of an even number if its first and last terms be of different signs, must have at least one real root.

7. Find the sum of the series

$1.2.4 + 2.3.5 + 3.4.6 + \&c.$ to n terms.

$\frac{1}{3} + \frac{1}{2} + \frac{3}{4} + \frac{9}{8} + \&c.$ to n terms.

and $1.2 + 2.3x + 3.4.x^2 + \&c.$ in inf.

8. If the force vary inversely as the square of the distance, and a body be projected at a given angle with a velocity which is to the velocity in a circle at the same distance, as $\sqrt{2} : 1$. Determine the nature of the orbit described.

9. Prove that the surface of any segment of a

sphere cut off by two parallel planes is to the whole surface of the sphere as the intercepted portion of the diameter is to the whole diameter.

10. Find the fluent of $\frac{(x-1)\cdot\dot{x}}{(x^2+1)^{\frac{3}{2}}}$; construct the fluent of $\frac{x\dot{x}}{x^2-2ax+1}$, where a is less than unity; and shew that the fluent of $\frac{d\dot{z}}{\sqrt{(a^2-bz^2)}} = \frac{d}{\sqrt{b}} \times$ circular arc whose sine is $\frac{z\sqrt{b}}{a}$ and radius $= 1$.

11. A paraboloid whose vertex is downwards is filled with water to a given altitude. Having given the diameter of the upper surface, find what ought to be the diameter of the hole at the bottom, so that the upper surface may descend through a given space in a given time.

12. If the force varies as the distance, and two bodies fall towards two different centres of force; compare their velocities at any points of their descent.

13. Two elastic balls, beginning to descend from different points in the same vertical line, impinge on a perfectly hard plane inclined at an angle of 45°, and move along a horizontal plane with the velocities acquired. Shew that if a circle be described, passing through the two points from which the balls began their motion, and touching the horizontal plane, the point of contact will bisect the dis-

tance between the vertical line and the point where they impinge on each other.

14. Given, that the distance of the centre of gravity of an area from its vertex is an (n^{th}) part of the abscissa; find the distance of the centre of gravity of the solid formed by the revolution of this area round its axis.

15. Determine the proportion between the radius of the globe and wheel, when the length of the cycloid within the globe (Sect. 10.) is a maximum.

16. If centripetal forces tend to the several points of spheres, proportional to the distances of those points from the attracted bodies, the compounded force with which two spheres will attract each other mutually is as the distance between the centres of the spheres.

Tuesday Evening.—Mr. Bland.

1. The sum of n arithmetic means between 1 and 19 is to the sum of the first $n-2$ of them :: 5 : 3. Find the means.

2. Two equal weights are connected by a string passing over a fixed pulley. Supposing a weight to be added on one side, and the length and weight of the string, and the difference of the altitudes of the weights at the commencement of the motion to be given; determine in what part of the descent, the

velocity will be neither increased nor diminished by the string's weight.

3. If the abscissa of a curve bear a finite ratio to the ordinate, prove that the abscissa will cut the curve in a finite angle.

4. The place of the node and the inclination of the moon's orbit to the plane of the ecliptic being given; find the place of the moon when her declination is the greatest possible.

5. Find the value of

$$\frac{\sqrt{(2a^3x-x^4)}-\sqrt{(ax^3)}}{a-\sqrt{(ax)}}, \text{ when } x=a;$$

and find the fluxion of

$$\text{the hyp. log. } \frac{\sqrt{(a+x)}+\sqrt{(a-x)}}{\sqrt{(a+x)}-\sqrt{(a-x)}}.$$

6. If, in a circle a straight line be drawn cutting the diameter at any angle (A); prove that the difference of the segments of the diameter will be to the difference of the segments of the line as the diameter is to the chord of an arc, which measures twice the complement of A.

7. If, from the extremity of the major axis of an ellipse which is perpendicular to the horizon, chords be drawn making with it angles of 75^0 and 45^0, and from the points where the chords meet the curve, ordinates be drawn to the axis; the square of the time down the first chord will be twice the square of the time down the second in the sub-

duplicate ratio of the rectangles under the segments of the axis made by the ordinates.

8. If a square be inscribed in a circle and another circumscribed about both: compare the pressures upon the circle and the squares when immersed vertically in a fluid; the angular point of the circumscribing square coinciding with the surface of the fluid.

9. A hollow cone, whose vertical angle is 60°, is filled with water, and placed with its base downwards. It is required to determine the place where a small orifice must be made in its side, so that the issuing fluid may strike the horizontal plane in a point, the distance of which from the bottom of the vessel is to the distance of the orifice from the top as 5 : 4.

10. The distance of the center of gravity of the surface of a solid from the vertex is equal to half the abscissa; determine the nature of the curve by the revolution of which round its axis the surface was generated.

11. If two equal parabolas be placed in such a manner that they may touch each other at the vertices, and one be made to roll upon the other, its focus will describe a right line; and the vertex cissoid, the diameter of whose generating circle is equal to half the latus rectum of the parabola.

12. If a body revolve in an orbit round a center

of force, and at the same time the orbit revolve round the same center in such a manner that the angular velocity of the body in the orbit if fixed, may be to its angular velocity when revolving, in the ratio of $F:G$; find the centripetal force necessary to retain the body in a revolving orbit, the force in the fixed orbit varying as the n^{th} power of the distance. And apply this to the cases of an ellipse when the center of force is in the focus, and when in the center.

13. Let α, β, γ, &c. be the roots of the equation $x^n - px^{n-1} + qx^{n-2} - \text{\&c.} = 0$, (m) of which are possible; shew that if the equation be transformed into one whose roots are $(\alpha-\beta)^2$, $(\alpha-\gamma)^2$, $(\beta-\gamma)^2$, &c. the last term of the transformed equation will be positive or negative according as $m.\dfrac{m-1}{2}$ is an even or an odd number.

14. Find the fluents of $\dfrac{e^z z \dot{z}}{(1+z)^2}$, where e = the base of hyp. logs;

$$\frac{dy^2\dot{y}}{(ar^2+by^2).\sqrt{(r^2-y^2)}}, \quad \frac{\dot{x}}{\sin.^2x \times \cos.^4x},$$

and
$$\frac{b\dot{y}}{(a^2-y^2).(a+y)^{\frac{2}{3}}}.$$

15. Find the relation of x to y in the equations,

$$y^n\dot{y} - a^{n-1}x\dot{y} + c^{n-1}y\dot{x} = 0,$$
$$a^2\dot{y}^2 + bx\dot{y}^2 = c^2\ddot{y},$$

and determine the nature of the curve, in which $\frac{\dot{x}}{x} - \frac{\dot{y}}{y} : \frac{\dot{x}}{y} - \frac{\dot{y}}{x} :: n : 1$, x and y being the abscissa and ordinate.

16. Find the sum of the series,

$$\frac{5}{1.2.3} + \frac{7}{2.3.4} + \frac{9}{3.4.5} + \frac{11}{4.5.6} + \&c.$$

to n terms by increments;

$$\frac{2}{3.5} - \frac{3}{5.7} + \frac{4}{7.9} - \frac{5}{9.11} + \&c. \text{ in inf.}$$

$$\frac{\text{tang. } A}{1} - \frac{\text{tang. } 2A}{2} + \frac{\text{tang. } 3A}{3} - \&c. \text{ in inf.}$$

and having given the sum of the series

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \&c. \text{ in inf.},$$

find the sum of

$$\frac{1}{1^2.2.3} + \frac{1}{2^2.3.4} + \frac{1}{3^3.4.5} + \&c. \text{ in inf.}$$

17. The reflecting curve is a semicircle, and the radiating point is in the circumference; determine the nature of the caustic, its length, and the density at different points.

18. The latitudes of two places on the earth's surface are complements to each other, and on a given day the sun rises (n) hours sooner at one place than at the other; determine the latitude of each place.

19. If a system of bodies be connected together and supported at any point which is not the center

of gravity, and then left to descend by that part of their weight which is not supported; $4l$ multiplied into the sum of all the products of each body into the space it has perpendicularly descended will be equal to the sum of all the products of each body into the square of its velocity; l being $= 16\frac{1}{12}$ feet.

20. A ring of given weight descends by its gravity down the arc of *any algebraic curve;* and the curve revolves uniformly about its axis which is perpendicular to the horizon in t''. Determine the velocity of the ring at any point of its descent.

21. If the force of gravity be uniform, and act perpendicularly to the horizon, determine the path of a projectile in a medium, the resistance of which is proportional to the velocity of the body.

22. Having given the relation between the centrifugal force and the force of gravity at the earth's equator; determine the relation between the centrifugal force and the force of gravity at the equator of Jupiter; the densities and times of revolution round their axes being known.

23. Shew that the mean motion of the nodes of the lunar orbit is not affected by the excentricity of the orbit; and that the true motion of the nodes in an elliptic orbit is equal to the motion of the nodes in a circular orbit when the radius vector is a mean proportional between the semi-axes major and minor.

24. If an hyperboloid and a cone be generated by the revolution of an hyperbola and its asymptotes; and the cone being excavated and placed with its axis vertical, water be poured into it till the surface touches the vertex of the hyperboloid; shew that whatever be the inclination of the axis, the surface of the water will always be a tangent plane to the hyperboloid.

1815.

Morning Problems.—Mr. Hustler.

MONDAY, JANUARY 16, 1815.

FIRST AND SECOND CLASSES.

1. If A be any arc whatever, prove that cosec. A + cosec. $2A$ + cosec. $4A$ + &c. to n terms $= \text{cot.}\,\frac{A}{2} - \text{cot.}\,2^{n-1}A$.

2. Shew that the sun's rising is least accelerated by refraction at the time of the equinoxes.

3. If an hyperbola and its asymptotes revolve about the axis major, prove that all the sections of the cone made by planes touching the hyperboloid have the same axis minor, which is the axis minor of the hyperbola.

4. Near the solstice the variations of the sun's declination are as the squares of the variations in longitude nearly.

5. Find from Taylor's Theorem the arc in terms of its *cosine*.

6. Having given the refracting powers of two mediums, find the ratio of the focal lengths of a convex and concave lens formed of these substances, which, when united, produce images nearly free from colour.

7. If t = tangent of half the angle ASP (Newton, Sect. 6.) to $r = 1$, shew that the parabolic area $ASP = a^2\left\{\frac{t^3}{3} + t\right\}$, where a is the focal distance of the vertex.

8. If a caustic be formed by a reflecting curve, shew that the reflected ray is always a tangent to the caustic.

9. In any recurring series $a + bx + cx^2 +$ &c. whose scale of relation is $f + g + h +$ &c., if any row of the differences of a, b, c &c. $= 0$, prove that $f + g + h +$ &c. $= 1$.

10. If a cylinder be cut by two parallel planes intercepting a given part of the axis, shew that the solidity between the planes is the same whatever be the inclination of the planes to the axis.

11. Find the fluent of

$$\frac{\dot{x}}{x^3(a^2 - x^2)}, \text{ and of } x^3\dot{x}\sqrt[3]{(a^2 + x^2)}.$$

12. Having given the latitude of the place and the moon's declination, determine the height of the superior and inferior tide; and compare the height

of the tide at the equator and the pole of the earth, when the moon's declination is 30^0.

13. If the moon's orbit be considered circular, and the position of the nodes be given, shew that, when the *horary* motion of the nodes is a maximum, the moon's distance from quadratures equals half the node's distance.

14. A semicubal parabola moves in its own plane, with its axis always coinciding with the same line. Determine the nature of a curve which, beginning at the vertex of the parabola in its first position, is perpendicular to it in all positions.

Monday Afternoon.—Mr. Hustler.

FIFTH AND SIXTH CLASSES.

1. If four quantities be in geometric progression, the sum of the two extremes is greater than the sum of the two means.

2. From the equation $a^{mx} = b - a^{mx-1}$, find the value of x.

3. If the interior angle BAC and the exterior angle DAC of any triangle ABC be bisected by lines AE, AF, which also cut BC in E, F, shew that BF, BC, BE are in harmonic progression.

4. Required the number of guineas with which

four persons *A*, *B*, *C*, *D* respectively begin to play, if after *A* has won half of *B*'s, *B* a third of *C*'s, and *C* a fourth of *D*'s, each has twelve guineas.

5. What power acting parallel to the length of an inclined plane whose elevation is 30^0, will draw a given weight Q, 40 feet up the inclined plane in $5''$?

6. The latus rectum of an ellipse, being produced both ways to meet the circle described on the axis major, = axis minor.

7. Given the cot. A and cot. B, find cot. $(A \pm B)$ radius being $= 1$; and adapt the expression to radius r.

8. Form the biquadratic equation, two of whose roots are $1 + \sqrt{(a^3)}$ and $-\sqrt{(-b)}$.

9. How far must a body fall internally towards the focus of an hyperbola to acquire the velocity in the curve?

10. The sun's altitude at any time being 30^0, find the position of a stick of given length so that the shadow may be the longest possible; and find the length of the shadow at that time.

11. Find the solid traced out by a curve whose equation is $y^2 = \dfrac{b^2 x}{a - x}$.

12. If a billiard-table be in the form of an ellipse, and a perfectly elastic ball be struck from either

focus in any direction, it will return after two reflections from the curve to the same point.

13. An object at the bottom of a vessel appears to change its place when water is poured into the vessel. Explain this circumstance.

14. Given the specific gravities of wood and water :: 2 : 3, to what depth would a given paraboloid of wood sink when immersed with its vertex downwards?

15. Find the fluxions of

$$\frac{a+x}{a^2+x^2}, \text{ of } x.(a^2+x^2)^{\frac{1}{2}}.\sqrt{(a^2-x^2)},$$

and of the secant of x; also the fluents of

$$\frac{\dot{x}\sqrt{(a^2-x^2)}}{x}, \text{ and of } \frac{b\dot{x}}{\sqrt{(1-ax^2)}}.$$

16. At a given place, and on a given day, find the point of the horizon where the sun rises.

17. Shew, according to Newton's second section, that if a parabola be described by a force tending parallel to the axis to a point indefinitely distant, the force must be constant.

Monday Afternoon.—Mr. Bland.

THIRD AND FOURTH CLASSES.

1. Extract the square root of $ab-d^2+4c^2\pm 2\sqrt{(4abc^2-abd^2)}$.

2. The sum of an even number of terms of any arithmetic progression, whose common difference is equal to the least term, will be four times the sum of half that number of terms diminished by half the last term.

3. The greatest and least corrected zenith distances of a circumpolar star being $38^0.\ 19'.\ 43''$, and $34^0.\ 53'.\ 49''$; determine the latitude of the place of observation.

4. Two forces acting upon a body in the same or in opposite directions, will cause it to move with a velocity equal to the sum or difference of the velocities which it would have received from the forces separately. Required a proof.

5. If bodies fall towards different centres of force, from different altitudes, compare the times of descending through any space; $F \propto \frac{1}{(\text{dist.})^2}$.

6. The length of the subtangent to the cissoid being equal to one fourth of the diameter of the generating circle; determine the point in the curve from which the tangent is drawn.

7. Transform the equation $x^3 - px^2 + qx - r = 0$ into one whose roots shall be mean proportionals between the roots of the equation, and a given quantity (m).

8. One side of a cubical vessel of water of given dimensions being loose; find the position, magni-

tude, and direction, of a single force which shall keep it at rest.

9. Find the time in which a vessel formed by the revolution of a given logarithmic curve round its asymptote, will empty itself through a given orifice in the bottom; the length of the axis and extreme ordinate being known.

10. If the force of gravity $\propto \frac{1}{(\text{dist.})^3}$, from the centre; find, according to Newton's method, the absolute velocity in feet, and absolute time in seconds, of descending through any space towards the centre.

11. Find the fluents of

$$z\dot{z}\sqrt{\frac{a+z}{a-z}}, \quad \text{and } \frac{\dot{z}}{z}.(a^2+z^2)^{\frac{1}{2}};$$

and shew that the fluent of

$$\frac{\dot{z}}{(1-az^2).\sqrt{(1-z^2)}} = \frac{1}{\sqrt{(1-a)}} \times \text{circular arc};$$

whose radius $= 1$, and cosine $=\sqrt{\frac{1-z^2}{1-az^2}}$.

12. The focal length of a double convex lens, whose thickness is inconsiderable, and whose surfaces have the same curvature, is equal to the diameter of one of the surfaces. Determine the ratio of the sines of incidence and refraction.

13. The areas of unequal ellipses are in a ratio compounded of the subduplicate ratio of their para-

meters and the sesquiplicate ratio of their principal axes. Required a proof.

14. If a body be acted upon by two forces which vary according to different laws of its distance from the centre, as the p^{th} and q^{th} powers; determine the angle described, while it passes from one apse to the other; the orbit described being nearly circular, and the forces at the apse being as $1 : n$.

15. Let a plane isosceles triangle vibrate edge-ways, suspended by its vertex. At what distance from its vertex must it strike an immoveable obstacle, so that its motion in the plane of vibration may be destroyed?

16. If two similar mediums are separated from each other by a space terminated on each side by parallel planes; and a body in its transit through this space, is attracted or impelled perpendicularly towards either medium, and is not agitated or hindered by any other force; and the attraction is every where the same at equal distances from either plane, taken towards the same side of the plane; prove that the velocity of the body before incidence is to its velocity after emergence as the sine of emergence to the sine of incidence.

Monday Evening.—Mr. Bland.

1. If (a) and (b) be the two first terms of a decreasing geometric progression, the sum of the series in inf. is $= \frac{a^2}{a-b}$. Required a proof.

2. If a tangent be drawn to any point of an ellipse, and from the point of contact a straight line be drawn to either focus; this shall be parallel to the straight line drawn from the centre to the intersection of the tangent and perpendicular from the other focus.

3. The moon's longitude at noon at Greenwich, January 1815, is

on the 16th. . . . 0°, $16'$, $48''$.
17th. . . . 12°, $46'$, $55''$.
18th. . . . 25°, $35'$, $31''$.
19th. . . . 38°, $46'$, $4''$.

Find its longitude on the 17th, at six o'clock.

4. In a given latitude, determine the vertical circle in which the difference of the altitudes of the sun in any two given days shall be a maximum.

5. If a body revolve in a reciprocal spiral, the force tending to the centre; prove that the times of its moving through successive angles of 180°, are in the proportion of the numbers $\frac{1}{1.2}$, $\frac{1}{2.3}$, $\frac{1}{3.4}$, &c.

6. Prove that every odd cube number is equal to the sum of as many terms of a series, which have a common difference unity, as its root contains units, the middle term of the series being the square of the root.

7. Find the value of

$$\frac{1}{x+a}+\frac{a}{(x+a).(x+b)}+\frac{ab}{(x+a).(x+b).(x+c)}+\text{\&c.}$$

to n terms; and of

$$\frac{4}{1.3}-\frac{12}{5.7}+\frac{20}{9.11}-\frac{28}{13.15}+\text{\&c. in inf.}$$

8. On the side of a vessel filled with fluid, let any number of circles be so situated that the pressures on them may be as the cubes of their diameters; determine the ratio of their distances from the surface of the fluid.

9. If water ascend and descend in the erect legs of a cylindrical canal, alternately; determine the nature and dimensions of the curve described by the centre of gravity of the water in the legs.

10. Two chains of the same uniform thickness and density are suspended from two given points, and attracted towards a centre of force, the law of the force being any power or root of the distance. Shew that the pressures on the points of suspension are proportional to the squares of the velocities which would be acquired by bodies falling towards the centre from the points of suspension, down spaces which are equal to the length of the chains.

11. Trace the curve whose equation is $x^2+y^2=\frac{b^2x^2}{2ax-x^2}$, and find its area when $b=a$.

12. A perfectly elastic ball A falls from the upper extremity of a given vertical line AB, and at the same time another perfectly elastic ball B is projected upwards from a horizontal hard plane at the bottom; they meet in some point C, and are reflected back. Determine the point C, so that they may ascend and descend from it continually; and find the velocity of B at that point.

13. Find the fluents of

$$e^x.(\dot{P}+P\dot{x}), \text{ of } \frac{dz^{pn-1}\dot{z}}{(a+cz^n)^m.(e+fz^n)^r}, \text{ and of}$$

$$\frac{x^{m-1}\dot{x}}{(1-x^m).\sqrt[2m]{(2x^m-1)}}.$$

14. If the middle points of any two edges of a triangular pyramid which do not meet, be joined; shew that the middle point of the connecting line is the centre of gravity of the pyramid.

15. If parallel rays be incident on a spherical surface of a plano-convex glass mirror, whose thickness is a semi-diameter and a half of the spherical surface, prove that they will, after having been refracted at the convex and reflected at the plane surface, converge to that point where the axis intersects the convex surface.

16. Two plane reflectors being inclined to each other at a given angle, determine the diameter of a

circular arc, in which a luminous object must move between them, so that the ray, which has been reflected by any given point of one of them, may after reflection at the second plane, always intersect the arc in the point in which the object is.

17. If the circle of curvature to the vertex of a parabola be described, and another circle touch that, and the arms of a parabola; and so on continually; prove that the radii of the circles will be in the proportion of the numbers, 1, 3, 5, 7, 9, &c.

18. If the force vary according to any law of the distance; shew that in any orbit, at the point where the centripetal and centrifugal forces are equal, the velocity towards the centre of force is a maximum.

19. Determine the nature of the curve by the revolution of which round its axis a solid will be generated; such that a corpuscle placed on its surface will be attracted towards the centre of gravity with a force varying as the distance; the solid revolving round its axis in a given time.

20. Find the horary increment of the area, which the moon, by a radius drawn to the earth, describes in a circular orbit. (Newton, Book III. Prop. 26.)

21. The excentricity of P's orbit (Sect. XI.) being small; find the variation of the major axis in a whole revolution of P, if the force at P be augmented or diminished by a small quantity in the ratio of $1 : 1 \pm n$.

22. If the co-latitude of the place of observation be equal to the moon's declination, or less than it, there will be no inferior or no superior tide, according as the latitude and moon's declination are of the same or different denominations.

23. Let a spherical body descend from rest in a fluid whose specific gravity is to that of the body as $1 : n$. Determine the velocity of the sphere at any point of its descent; and shew that the greatest velocity which it can acquire is equal to that which would be acquired by it in descending from rest, in vacuo, by the constant force of its comparative gravity through a space which is to $\frac{4}{3}$ of its diameter $:: n : 1$.

24. A given cylindrical rod falls by gravity towards a horizontal plane, whilst at the same time its extremity moves freely along the plane. Determine the pressure upon the plane in any position, and the velocity of the moving point.

Tuesday Morning.—Mr. Bland.

FIRST AND SECOND CLASSES.

1. If the terms of the series arising from the expansion of $(a+b)^n$ be multiplied respectively by the terms of the arithmetic series $0x$, $1x$, $2x$, $3x$, &c. find the sum of the resulting series.

2. If a polygon has $(n+4)$ sides; prove that the angles formed at the points of concourse of

these sides produced are together equal to $(2n)$ right angles.

3. If a body move in a curve round a centre of force, and the force by which it is retained in the curve vary in a less ratio than the inverse cube of the distance, prove that the body cannot fall into the centre.

4. Shew that in the spiral, where the angle described by the radius vector $SP \propto SP$ directly; the number of revolutions which have been made by it varies as the square root of the subtangent to the point P.

5. Transform the equation $2x^3 - 2x^2 + 3x + 6 = 0$ into one which shall have its signs alternately positive and negative.

6. Two bodies A and B move in opposite directions with velocities, the sum of which is given. Shew that the sum of the products of each body into the square of its velocity is a *minimum*, when the velocities are reciprocally proportional to the quantities of matter in the bodies.

7. If from one extremity of the diameter of a circle, chords are drawn intersecting the radius which is perpendicular to the diameter or that radius produced, and from the points of intersection ordinates be erected, always equal to the cosine of the arc measured from the opposite extremity of the diameter to the chord; determine the nature of the curve which is the locus of the ordinates.

8. Find the fluent of $\dfrac{d z^{\frac{1}{3}} \dot{z}}{(c^{\frac{5}{3}} - a z^{\frac{2}{3}})^{\frac{3}{5}}}$, and of $\dot{x} \int \dot{x} \int \dot{x} \int \dot{x}$ in infinitum; and find the relation of the abscissa and ordinate of a curve, when $\epsilon^{z} = \epsilon^{x} - \epsilon^{-x}$: ϵ being the base of hyp. log., and z and x the arc and abscissa respectively.

9. When parallel rays are incident upon a spherical reflector, shew that the radius of the least circle of aberration varies directly as the cube of the semi-aperture, and inversely as the square of the focal length of the reflector.

10. A particle P is attracted to a sphere by forces $\propto \frac{1}{D^4}$. If on the line joining P and the centre of the sphere a semi-circle be described and made to revolve; it would cut out a portion of the sphere, the attraction to which is equal to the attraction of the remaining part.

11. Find the sum of the series cos. $A + \frac{1}{2}$ cos. $2A + \frac{1}{3}$ cos. $3A + \frac{1}{4}$ cos. $4A$ + &c. and resolve $1 + 5 + 19 + 65 + 211 +$ &c. whose scale of relation is $f - g$, into two geometric series whose corresponding terms added together will give the proposed series.

12. Prove that the mean tides are equally affected by the northerly and southerly declinations of the moon.

13. The quadrant of a circle is impelled by a

stream in its own plane in the direction of the extreme radius. Find the direction in which it will begin to move.

14. Find the equation to the section of a solid generated by the revolution of a given algebraical curve about its axis.

Tuesday Afternoon.—Mr. Bland.

FIFTH AND SIXTH CLASSES.

1. Prove that the opposite sides of an equilateral and equiangular hexagon are parallel.

2. Determine the roots of the equation $x^4 - 4x^3\sqrt[4]{2} + 6x^2\sqrt[4]{4} - 4x\sqrt[4]{8} + 2 = 0$; and find the equation whose roots are $\frac{1}{2}a + \sqrt{(-\frac{3}{4}a^2)}$; $\frac{1}{2}a - \sqrt{(-\frac{3}{4}a^2)}$; $-a$.

3. If two equal forces, inclined at any angle, act upon a body, prove that the compound force bisects that angle.

4. Given the meridian altitudes of the upper and lower limb of the sun $18^\circ. 39'. 39''$, and $18^\circ. 7'. 3''$. Determine its diameter, and the altitude of its centre.

5. Find the sum of the series,

$6 + 2 - 2 - 6 -$ &c. to 19 terms;

$8 + 20 + 50 + 125$ to 15 terms;

$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} +$ &c. to n terms.

6. If a body revolve in a circular orbit about the earth at a distance from its surface equal to 20 radii of the earth; what is the measure of the subtense of the arc described in $1''$?

7. If from the extremity of the diameter of a circle a straight line be drawn touching the circle and equal in length to the circumference, and a triangle be formed by joining its extremity and the centre. Prove, that if from any point in this line a perpendicular be drawn to the base, the circumference of the circle described with this as radius, shall be equal to the part of the base intercepted between the perpendicular and the acute angle.

8. The equation $x^4 - \frac{1}{2}x + \frac{3}{16} = 0$ has two equal roots. Find all the roots.

9. Prove that the periodic times of bodies revolving in different ellipses round different centres of force in the foci are in the sesquiplicate ratio of their major axes directly and the subduplicate ratio of the forces inversely.

10. Construct Newton's telescope; and find the magnifying power, and greatest field of view.

11. Find the fluents of

$$\frac{\dot{x}}{x^2\sqrt{(x^2-a^2)}}, \quad \frac{\dot{x}}{x^3-2ax^2+x},$$

where a is less than unity: and the n^{th} fluxion of $\sqrt{x}$.

12. If a body move in a conic section acted upon

by a force tending to the focus S, shew that the velocity at the distance SP is to the velocity at any other distance SQ as a mean proportional between HP and SQ is to a mean proportional between SP and HQ, H being the other focus.

13. Determine the arc of a given circle, the rectangle under whose sine and excess of sine above the cosine is a maximum.

14. The radius of a circle whose area is equal to the surface of a given cone is a mean proportional between the side of the cone and the radius of its base. Required a proof.

15. Compare the absolute forces in the centre and circumference of a circle, so that the periodic times may be the same.

16. An (n^{th}) part of a hollow paraboloid with its vertex downwards is filled with a fluid of known specific gravity; and a sphere of given size and substance is immersed. Find how high the fluid will rise.

Tuesday Afternoon.—Mr. Hustler.

THIRD AND FOURTH CLASSES.

1. If the two sides of a spherical triangle together $= 180^\circ$, the arc which bisects the vertical angle, bisects the base also.

2. One root of the equation $x^4+x^3-8x^2-16x-8=0$, is $1-\sqrt{(5)}$. Find the other roots

3. Two bodies are projected towards each other in the same vertical plane from two given points, so as to describe the same parabola. Find the point where they meet.

4. Given the right ascension and declination of a star, and the latitude of the place; determine the day of the year when the star rises the same instant with the sun.

5. In the parabola, the normal is the least line which can be drawn from a given point in the axis to the curve.

6. Required the fluxion of $x \times \epsilon^{\tan. x}$, and the following fluents,

$$\int \frac{\dot{x}}{x^3 \sqrt{(1-x^2)}}, \quad \int \frac{b x^{\frac{1}{2}} \dot{x}}{\sqrt{(a^7 - x^7)}}, \quad \int \frac{\dot{x} \text{ h. l. } x}{x}.$$

Shew also that $\int \dot{x} \text{ cosec. } 2x = \frac{1}{2} \text{ h. l. tan. } x$.

7. If the bases of a cylinder, and of a cone, have the same radius as a sphere, and each of their altitudes = the diameter of the sphere, the solidity of the cone = the excess of the cylinder above the sphere.

8. A body revolving in an ellipse at the mean distance, is projected perpendicularly to the distance with the velocity with which it is there moving. Shew that it describes a circle, and in the same periodic time in which it would have described the ellipse.

9. The equation to a curve is $y^m = a x^{m-1} + x^m$;

draw its asymptote, and determine the angle which it makes with the axis.

10. Compare the time of emptying a cone and its circumscribing cylinder; the vertex of the cone being downwards and coinciding with the orifice.

11. If an object be seen through a double convex lens, determine the proportion in which it is magnified, when the distances of the eye and object from the center are each equal to half the focal length.

12. Sum the following series,

$$\frac{1}{1.5}+\frac{1}{3.7}+\frac{1}{5.9} \text{ to } n \text{ terms and ad inf.}$$

$$1.4+2.5+4.6+\text{\&c. to } n \text{ terms,}$$

$$\frac{1}{1.2}-\frac{2}{2.3}+\frac{3}{3.4}-\text{\&c. ad inf.}$$

13. Suppose two bodies fall towards a center of force, one acted upon by a force varying as the distance, and the other by a constant force which is half the variable force at the beginning of the motion. Shew that the velocities acquired at the center will be equal.

14. Give Cotes's construction of the elliptic spiral, and shew in what cases it cuts itself.

15. In the ordinate PN of an ellipse, whose center is C, a point Q is taken so that CQ always $=PN$. What is the curve which passes through the points Q?

16. If a body describes an epicycloid, the force tending to the center of the globe, required the law of the force.

17. If p and q be two weights applied at the circumferences of a wheel and axle, find the proportion between the radii, so that the time of q ascending through a given space may be a minimum, the inertia of the wheel and axle being considered.

Tuesday Evening.—Mr. Hustler.

1. A pays P£. to B, on condition that he receives an annuity during the life of an individual, who, according to the tables, may be expected to live n years. What must be the annuity?

2. In an unlimited problem, $mx+ny=p$; if m and n both measure p, m is the least integral value of y, and n of x.

4. A cylinder of indefinite length is placed before a convex reflector, and their axes coincide. Shew that its image is a cone, whose vertex is the principal focus of the reflector.

4. Determine the situation of a fixed star, so that its right ascension may be unaffected by the precession of the equinoxes.

5. Investigate Taylor's Theorem, by the method of differences.

6. A sphere of given radius is suspended in the air. At a given place, day, and hour, determine

the figure of its shadow on a horizontal plane; and shew that the length of the shadow varies as the secant of the sun's zenith distance.

7. Shew that the variation of the radius of curvature of any meridian of the earth, is as the square of the sine of latitude.

8. Prove that the value of a vanishing fraction $\frac{P}{Q}$ may be found by taking successively, if necessary, the first, second, and third, &c. fluxions of the numerator and the denominator; and investigate the method of finding the value of such a fraction when the indices are fractional.

9. *TB*, *BC* are the subtangent and ordinate of a curve whose vertex is *A*, and the tangent of the angle *TCA* is to the tangent of the angle *ACB* in a given ratio. What is the nature of the curve?

10. For any position of the line of the nodes, construct for the inclination of the moon's orbit to the plane of the ecliptic. (Newton, vol. III. Prop. xxxv.)

11. *ABCD* is a section of a four-sided glass prism perpendicular to its axis, having one angle $D=90^\circ$, and the opposite angle $B=135^\circ$. Shew that a ray of light entering the prism perpendicularly to *AD*, and reflected by *AB*, *BC*, will pass through *CD* without refraction, and thence explain the *Camera lucida*.

12. A paraboloid has its axis parallel to the

horizon, and a flexible chain is wound round its greatest circular section; find the length which will be unwound after t'' have elapsed, a given part being unwound at the beginning.

13. Two equal distances CA, CB are drawn at right angles to each other from a centre of force C. In CA any point D is taken, and DE is drawn to CB so that DE may equal CB or CA. Two perfectly elastic balls fall from A and B at the same time, the force varying as the distance, and are reflected at D and E by two planes inclined to their motion at $\angle\ 45^\circ$. Investigate their subsequent motions.

14. Find the fluents of

$$\frac{\dot{x}}{x^5\sqrt{(a^2-x^2)}}, \quad \text{of } \frac{\dot{x}\sqrt{(1+x)}}{(1-x)^{\frac{3}{2}}},$$

$$\text{and of } \frac{\dot{x}.(1-x^2)}{(1+x^2)\sqrt{(1+ax^2+x^4)}}.$$

15. Find the relation between x and y, when $\ddot{y}\sqrt{(ay)}=\dot{x}^2$; and shew that

$$\int\frac{x^{p-1}\dot{x}}{\sqrt{(1-x^n)^{n-q}}}=\int\frac{x^{q-1}\dot{x}}{\sqrt{(1-x^n)^{n-p}}},$$

between the values of $x=0$, and $x=1$.

16. ACB is a quadrant, and one extremity D of a line CD which equals its chord AB, moves along the radius OB produced, while the other extremity C moves in the periphery of the quadrant. Find the equation and area of the curve described by a point P in the middle of the line CD.

17. If m be any prime number, and x any other number prime to m. Then x and x^m being severally divided by m, leave the same remainder.

18. Sum the series,

$$1 + \frac{2}{2.3} + \frac{2^2}{3.3^2} + \frac{2^3}{4.3^3} + \&c. \text{ ad inf.}$$

$$\frac{5}{1.2.3.4} + \frac{9}{3.4.5.6} + \frac{13}{5.6.7.8} + \&c. \text{ to } n \text{ terms,}$$

and ad inf. by the method of increments.

19. A paraboloid floats in a fluid, the axis not being perpendicular to the horizon. Determine the position in which it rests; the specific gravities of the paraboloid and the fluid being given.

20. CP, CD, are two semi-conjugate diameters of an ellipse, whose center is C. EP, which is perpendicular to CD, is produced to L making PL equal to CD, and through K the middle point in CL, $MKPN$ is drawn so that KM and KN may each equal CK. Shew that the semi-axes major and minor equal PM and PN respectively, and determine their positions.

21. A person turning up three cards from a common pack, undertakes that the number of points upon them shall be either 29, 19, or 9, reckoning 11 or 1 for the ace, and 10 for each of the court cards. What are the odds against him?

22. If a body A be attracted towards two centres of force S and T, and be projected in a direction oblique to the plane STA, the solids generated by

the motion of the triangle joining *S*, *T*, and the body, shall be proportional to the times of description.

23. *A* and *B* are two ships at sea; *B* moves in a given straight line, and *A* endeavours to overtake *B* by always moving towards it. Having given the velocities of *A* and *B*, investigate the curve traced out by *A*.

1816.

Monday Morning —Mr. French.

Monday, January 15, 1816.

FIRST AND SECOND CLASSES.

1. Given the logarithms of 6 and 7, shew how the logarithm of 1767 may be computed.

2. If the digits composing any number, be inverted, the difference between the number, so formed, and the original number, is divisible by nine.

3. In the oblique impact of an imperfectly elastic body upon a plane, co-tan. incidence : co-tan. reflection :: force of compression : force of elasticity.

4. In Gregory's telescope, the aberrations, produced by the two reflections, are in the same direction.

5. At a given place, on a given day and hour, the sun's azimuth is double that of a known star; required the distance of the sun from the star.

6. An imperfectly elastic ball being projected from *P*, a point in the periphery of a circle *PQR*,

whose center is C, after impinging at Q and R returns to P; required the value of the angle CPQ.

7. Investigate an expression for the length of a caustic by reflection, and apply it, in the case of parallel rays incident upon a concave spherical reflector.

8. In the common pump, given the height of the fixed sucker above the surface of the reservoir, and the space through which the piston descends; required the altitude of the water in the tube after n strokes of the piston.

9. To determine the radius of curvature in the curve, whose ordinate is equal to the circular arc, of which its abscissa is the versed sine.

10. To find the sum of all the powers of the roots of an equation.

11. To draw a diameter to a curve of n dimensions. (M'Laurin's *Algebra.*)

12. Compare the curvatures of the moon's orbit in quadrature and syzygy; supposing that the orbit, independently of the sun's disturbing force, would have been a circle.

13. To determine the number of given points, through which a curve line of the m^{th} order, may be drawn.

Monday Afternoon.—Mr. French.

FIFTH AND SIXTH CLASSES.

1. What decimal of a pound is $11d. \frac{1}{2} \frac{1}{3}$?

2. Investigate the rule for finding the least common multiple of two quantities, and apply it to find the least common multiple of 177 and 2982.

3. Required to express the sum of the alternate terms of a binomial raised to the m^{th} power, beginning with the second.

4. If one side of a triangle be bisected, the sum of the squares of the other two sides is double of the square of half the line bisected, and of the square of the line drawn from the point of bisection to the opposite angle.

5. Compare the area of the hexagon inscribed in a given circle with the area of the circumscribing hexagon.

6. Given the figure of an ellipse, find practically its center and its foci.

7. In an ellipse, if the line Ioi be drawn parallel to the axis minor BCD, and Qoq, parallel to the axis major ACM; then $Io \times oi : Qo \times oq :: BC^2 : AC^2$. Required a proof.

8. Prove, geometrically, that in any plane triangle, the sum of the sides is to their difference as the tangent of half the sum of the angles at the base to the tangent of half their difference.

9. Shew that $(\tan.)^3\, 60^\circ = 3 \tan. 60^\circ$ to rad. $= 1$.

10. P and W being in equilibrio on an inclined plane, if the whole be put in motion, then P's velocity : W's velocity :: W : P.

11. A perfectly hard body is let fall from a given height (a) upon a half-elastic horizontal plane; required the height to which it will rise after impinging the third time upon the plane.

12. Compare the time down $\frac{1}{n}$th part of a given inclined plane with the time down the remainder.

13. Explain the principle of the syphon.

14. Determine the visual angle in Cassegrain's telescope.

15. Given the latitude of the place, and the altitude of the sun in the equinoctial, find the hour-angle.

16. The force tending to the focus of an hyperbola varies inversely as the square of the distance.

17. Required to express the fluxion of the arc in terms of the fluxion of the co-secant.

18. The roots of the equation $ay^3 - by^2 - cy + 1 = 0$, are in harmonic progression, find them.

19. Required the fluents of $\dfrac{mbx^{-n-1}\dot{x}}{\sqrt{(e+fx^{-n})}}$,

$$\frac{\sqrt{(y)}.\dot{y}}{1+y^{\frac{3}{2}}},\quad \frac{\dot{x}}{\sqrt{(x^2-a^2)}},\quad \frac{x^{\frac{1}{2}}\dot{x}}{\sqrt{(2a-x)}},\quad \frac{z\dot{z}}{\sqrt{(a^4-z^4)}}.$$

Monday Afternoon.—Mr. Bland.

THIRD AND FOURTH CLASSES.

1. Prove that the sectors of two different circles are equal, when their angles are inversely as the squares of the radii.

2. If a circle be described about the center of gravity of a system of bodies, A, B, C, &c., and any point S be taken in the circumference, shew that $A \times SA^2 + B \times SB^2 + C \times SC^2 +$ &c. is constant.

3. The radius of curvature of the common parabola has to the normal the duplicate ratio that the normal has to the semi-parameter. Required proof.

4. Find the center of gravity of the solid generated by a quadrant of a circle through one-fourth of a revolution about the radius.

5. Given (a) and (b) the $(m)^{th}$ and $(n)^{th}$ terms of an arithmetic progression; determine the value of the $(x)^{th}$ term.

6. Find the fluents of $\dfrac{d\dot{x}}{a^3 - mx^2}$, $\dfrac{d\dot{x}\sqrt{(x)}}{\sqrt{(1-x)}}$,

and $\dfrac{x^2\dot{x}}{(x-a)^2.(x+a)}$.

7. From a given right cone cut off a parabola such, that its area shall be double the rectangle contained by the segments of the diameter of the base, formed by the section.

8. If the force $\propto \frac{1}{D^2}$, and a body be projected in a given direction with a velocity which is to the velocity in a circle at the same distance in a less ratio than $\sqrt{2} : 1$; determine the nature of the orbit described.

9. If an object be placed between two plane reflectors inclined to each other at any angle, and the eyes of a spectator be in any point between the planes, the distance of the eye from any of the images seen by him, is equal to the length of the path described by the rays which form that image.

10. Prove that no fraction can be represented by a terminating decimal, unless the denominator be 2 or 5, or the product of some powers of 2 and 5.

11. The angle included between the hour-lines of 12 and 3, is equal to the angle included between the hour-lines of 4 and 6, in a horizontal dial. Determine the latitude for which the dial is constructed.

12. What must be the nature of a parabolic curve which revolving round its axis would generate a solid, such that the time of emptying it would be to the time of emptying the circumscribing cylinder in the ratio of 1 : 9.

13. If the attraction of the earth and moon be as their quantities of matter directly and the squares of their distances inversely; what is the nature of the curve in which a body being placed would be equally attracted to both?

14. Compare the ablatitious force with the mean force of P to T (11th Sect.); the periodic times of P and T being as $1 : n$.

15. Prove that if two bodies be projected in similar directions with velocities which are in the subduplicate ratio of the force and distance, they will proceed in similar curves.

Monday Evening.—Mr. Bland.

1. Given the sum (s) and the sum of the squares (S) of a geometric series continued in infinitum. Determine the series.

2. If the bases of two equal cycloids be parallel, and the vertex of one in the base of the other; prove that the angle formed by the intersection of the curves will be a right angle.

3. If two conic sections be described on the same axis major, and have the same abscissæ, the ordinates will be in the sub-duplicate ratio of the *latera recta*. Required proof.

4. In a system of wheels moveable by teeth and pinions, having given the ratios of the number of teeth in each wheel and pinion, determine the number of times the $(n)^{th}$ wheel turns round its axis while the first performs (m) revolutions.

5. Trace the curve whose equation is $a \,.\, (y-b)^2$

$=x.(x-a)^2$, and determine the angle at which the curve cuts the axis when $x=a$.

6. If the altitudes of the sun be taken at the same place on the same day, when he is in the same vertical in opposite directions; shew that the sum of their tangents will be to the sum of their secants, as the sine of the sun's declination is to the sine of the latitude of the place.

7. Prove that the sums of the reciprocals of the $(n)^{\text{th}}$ powers of the odd and even numbers are to each other in the proportion of $2^n - 1 : 1$.

8. Find the sum of the series:

$$\frac{2a+b}{a^2.(a+b)^2} + \frac{2a+3b}{(a+b)^2.(a+2b)^2} + \frac{2a+5b}{(a+2b)^2.(a+3b)^2} + \&c. \text{ in inf.}$$

$$\frac{1}{1.4.7} + \frac{1}{4.7.10} + \frac{1}{7.10.13} + \&c. \text{ to } n \text{ terms by}$$
the method of increments.

$$\text{and } \frac{1}{4.7.12} + \frac{1}{6.10.16} + \frac{1}{8.13.20} + \&c. \text{ in inf.}$$

9. Determine the inclination of a plane of given length, so that a cylinder of known dimensions and uniform density, may roll freely down it in a given time.

10. A barometer, having some air in the tube, stands at an altitude of (a) inches; but being put under the receiver of an air-pump which contains (n) times as much as its barrel, after m turns it

stands at an altitude (b). Find the standard altitude, and the quantity of air in the tube at first.

11. An aperture of given area is cut from the top to the bottom of the side of a regular vessel full of water. Required its nature and dimensions, such that the velocity of the descending surface may be as the $(n)^{th}$ power of its distance from the lowest point; the velocity of every particle of the issuing fluid being supposed to vary as the square root of its depth below the surface.

12. Shew that in the spiral where the angle described by the radius vector $SP \propto SP^m$, the areas described by SP in one, two, three,....n revolutions, measuring from the center, will be as the numbers, 1, 2, 3....n, raised to the power $\frac{m+2}{m}$.

13. Find the fluents of $\frac{(a+bx).d\dot{x}}{x^3-1}$;

$\frac{\dot{x}}{x^n\sqrt{(a^2-x^2)}}$, where n is an even number;

and $\frac{d\dot{x}}{x.(1+x)^2.(1+x+x^2)}$.

14. Two imperfectly elastic bodies A and B are at a given distance in the same vertical line; A, the higher is acted on by gravity, which is supposed to have no effect on B. Shew that if A fall and strike B successively, the intervals between the strokes decrease in geometric progression; and determine the space passed over after any number of strokes.

15. If a small pencil of diverging rays be incident nearly perpendicularly on the spherical surface of a plane convex glass mirror, the radius of which is known; determine what must be its thickness, so that after refraction at the convex, and reflection at the plane surface, they may converge to that point where the axis intersects the spherical surface; the focus of incidence being given, and at a greater distance from the surface than the diameter of the sphere.

16. Determine the nature of the curve which will refract parallel rays to or from one focus, when the cosine of incidence is to the sine of refraction :: $1 + n : 1$.

17. Given the greatest and least apparent diameters of the moon, find what would be the apparent diameter corresponding to the mean distance; and shew that it is less than the mean apparent diameter.

18. If any number of bodies be retained in horizontal circular orbits by means of strings of unequal lengths, and the distances of the centers from the points of suspension be equal, the times of their revolutions will be the same.

19. Determine the nature and area of a curve such that if a right line be drawn from its vertex making an angle of 45^0 with the axis, the portion of the ordinate intercepted between this line and the curve shall always have to the sub-tangent the

inverse ratio that the ordinate has to the given line (a).

20. Let a sphere of a given diameter be projected in a fluid, the specific gravity of which is to that of the sphere as $1 : n$; having given the velocity of projection, determine what part of it is lost during the time the body describes any given space.

21. Shew that the effect of the sun upon the matter of the earth exterior to the inscribed sphere, to turn it about its center, is equal to the effect which would be produced if one-fifth part of that matter was placed at that point of the earth's equator which is opposite to the sun.

22. Required the area of the rhumb-line considered as a spiral; and shew that its orthographic projection on the plane of the equator is an hyperbolic spiral.

23. If the particles of air be moved from their places by a force which varies according to any given law; it is required to find the law of the force with which they will *continue* to be agitated, supposing the elasticity of the atmosphere to be proportional to its density.

24. If a chain of given weight reaching to the center of the earth be suspended from a cylinder at the surface, round which it is made to wind itself by the descent of a weight (w) unwinding a string supposed to be without weight; determine the

velocity of w at any point, and also where it is the greatest.

Tuesday Morning.—Mr. BLAND.

FIRST AND SECOND CLASSES.

1. If a series of arcs be taken in arithmetic progression, the radius of the circle will be to twice the cosine of the common difference as the cosine of any arc taken as a mean is to the sum of the cosines of any two equidistant extremes.

2. Find the sum of a recurring decimal $qppp$ &c. in inf., (q) and (p) containing (m) and (n) digits respectively.

3. In the direct impact of perfectly hard bodies, the difference between the sums of the products of each body into the square of its velocity before and after impact, is equal to the sum of the product of each body into the square of the velocity gained or lost.

4. If S, a, r, n be respectively the sum, first term, common ratio and number of terms of a geometric progression; find the sum of the series,

$$(S+a)+(S+a+ar)+(S+a+ar+ar^2)+\&c.$$

5. Prove that if contiguous and parallel rays of light fall upon a refracting sphere, the homogeneal rays will emerge parallel after n reflections and two refractions, when the least cotemporary variations of

the angles of incidence and refraction are to each other as $n+1 : 1$.

6. If an equilateral triangle be inscribed in a circle, and the adjacent arcs, cut off by two of its sides, be bisected; the line joining the points of bisection will be trisected by the sides.

7. The sum of the distances of a star from two known stars is a minimum, and its declination, which is greater or less than that of each of the others is known; determine its right ascension.

8. Let $y = A+Bx+Cx^2+Dx^3 + \&c.$ where A, B, C, D, &c. are constant quantities; then if $[y]$, $\left[\frac{\dot{y}}{\dot{x}}\right]$, $\left[\frac{\ddot{y}}{\dot{x}^2}\right]$, &c. be the values of y, $\frac{\dot{y}}{\dot{x}}$, $\frac{\ddot{y}}{\dot{x}^2}$, &c. when $x=0$; prove that

$$y=[y]+\left[\frac{\dot{y}}{\dot{x}}\right]\cdot\frac{x}{1}+\left[\frac{\ddot{y}}{\dot{x}^2}\right]\cdot\frac{x^2}{1.2}+$$
$$\left[\frac{\dddot{y}}{\dot{x}^3}\right]\cdot\frac{x^3}{1.2.3}+\&c.$$

9. Find the fluent of $\dfrac{\dot{z}}{z\sqrt{(a+cz^n)}}$;

and of $\dfrac{x^p\dot{x}}{(1+x^n)^2}$,

the fluent of $\dfrac{x^p\dot{x}}{1+x^n}$ being $=A$.

10. The velocity of a body descending from an infinite distance towards a centre of force, is $\frac{1}{n}$th

part of the velocity in a circle at the distance of that point; it is required to determine the law of the force.

11. If P be the place of a comet in its parabolic orbit, and a circle be described through P, the vertex and the focus; shew that the time of moving from perihelion to P will be proportional to the perpendicular drawn from the center of the circle to the axis.

12. If $ay^m\dot{y} = cx^n\dot{y} - ayx^{n-1}\dot{x}$; determine the algebraic relation of x and y.

13. In latitude 45° the mean altitude of the tide is always the same whatever be the declination of the moon.

14. From two bags, one of which contains (m) and the other (n) balls, marked a, b, c, d, &c. (m) being greater than (n), two balls are drawn; what is the probability that they have both the same letter?

Tuesday Afternoon.—Mr. Bland.

FIFTH AND SIXTH CLASSES.

1. Extract the square root of $a + x + \surd(2ax + x^2)$.

2. The sum of (n) terms of any arithmetic progression whose common difference is equal to the least term, will be equal to the sum of $(n+1)$ mag-

nitudes, each of which is half the greatest term of the progression.

3. If four quantities of the same kind be proportional; the first shall have to the third the same ratio that the second has to the fourth. (Euc. B. 5.)

4. If $A \propto B$, and $B \propto C$; prove that $A \propto mB \pm nC$, where (m) and (n) are known quantities.

5. In the steel-yard, if the weight increase in arithmetic progression, the divisions of the scale will be at equal intervals; and if each of these intervals be equal to the shorter arm, the moveable weight will be equal to the difference of the arithmetic progressionals.

6. Two bodies descend, one vertically through 400 feet, and the other down an inclined plane 500 feet long, and inclined at an angle of 30^0 to the horizon, compare the times of their descents.

7. The solid content of a cone whose base is equal to a great circle of a sphere, and altitude equal to the diameter, is half the solid content of the sphere.

8. If the force $\propto \frac{1}{D^2}$, determine how far a body must fall externally to acquire the velocity in the ellipse.

9. Find the fluents of

$$\frac{(a+bx).\dot{x}}{a^2+x^2}, \text{ and } \frac{x^4\dot{x}}{\sqrt{(a^2+x^2)}}.$$

10. Construct Newton's telescope, investigate its magnifying power, and find the linear magnitude of the greatest field of view.

11. If a vertical straight line be placed before a plane mirror inclined at an angle of 45° to the horizon, determine the image and its position.

12. Given $\sqrt{\frac{\{(x+a)^3\}}{x-a}}$ = a minimum. Find the value of x.

13. A fluid issuing from the side of a vessel (h) feet high, struck the horizontal plane at a distance of (d) feet from the bottom. Determine the point in the side of the vessel where the orifice is made.

14. If the force $\propto \frac{1}{D^2}$, and bodies fall from different altitudes towards different centers of force, determine the proportions of the times in which they fall through any space.

15. If the force $\propto \frac{bA^m + cA^n}{A^3}$; find the angle between the apsides.

16. If A, A', A'', represent the areas of three similar rectilineal polygons described on the hypothenuse and sides of a right-angled triangle, $A = A' + A''$. Required a proof.

Tuesday Afternoon.—Mr. French.

THIRD AND FOURTH CLASSES.

1. Prove, geometrically, that $1 + \cos. 2\theta = 2\cos.^2 \theta$.

2. A perfectly elastic ball, projected from A directly up an inclined plane, AB, strikes a vertical plane passing through B and returns to A; required its velocity at B, the length of the plane being 36 feet, and its elevation 30°.

3. In an hemispheroid emptying itself by a small orifice in the vertex, compare the time in which the surface of the fluid descends through the upper half of its axis, with the time through the lower.

4. To find the variation of the angle, which a given object subtends at the eye when viewed through a convex lens, the object being farther from the lens than its principal focus (F), and the eye nearer to the lens than its principal focus (f).

5. The precession in right ascension is positive when the angle of position is acute, and negative when it is obtuse. Required a proof.

6. The least error in time due to a given error in the altitude of a known star being b''; to determine the latitude of the place, and the true zenith-distance of the star.

7. To find the area of the conchoid of Nicomedes.

8. If the moveable orbit be projected *in antecedentia*, with a velocity equal to that of *P in consequentia*, shew that the velocity of *p* vanishes, when *Cp* becomes the least distance in the ellipse.

9. Investigate the equation between the perpendicular and the distance, in the lituus $\left(\angle \propto \frac{1}{\text{dist.}^2}\right)$ and determine the point of contrary flexure in the curve.

10. The roots of the equation, $y^4 - 8y^3 + 14y^2 + 8y - 15 = 0$, are in arithmetical progression. Find them.

11. Required the fluxion of the arc whose sine $= 2y\sqrt{(1-y^2)}$.

12. Apply Newton's method of making a body oscillate in a cycloid, to the common cycloid.

13. Find the following fluents:

$$\int \frac{x^4 \dot{x}}{\sqrt{(a^2-x^2)}}, \quad \int \frac{2a\dot{x}}{x\sqrt{(a^3+x^3)}}, \quad \int \dot{\theta} \cos.^3 \theta.$$

14. Sum the following series:

$$\frac{1}{1.3} - \frac{1}{3.5} \text{ \&c. in inf.}$$

$$1 + 3 + 7 + 15 + \text{\&c. to } n \text{ terms.}$$

$$\frac{1}{1.5.9} + \frac{1}{5.9.13} + \text{\&c. to } n \text{ terms by increments.}$$

Tuesday Evening.—Mr. French.

1. If a sum p, at compound interest, in n years amounts to m, in what time will the same sum amount to M, at the same rate?

2. A cylindrical vessel, of given thickness, is required to be of a certain capacity, find the least quantity of material with which it can be made.

3. A thin rod, formed of the arcs of two entire, unequal cycloids, lying in the same plane, and on opposite sides of the line of their bases, floats upon a fluid, sinking to the point at which the arcs are united; to determine its position when at rest.

4. To find the surface of a sphere by the method of indivisibles.

5. Required the time of oscillation in a finite circular arc.

6. To determine that point in the periphery of an ellipse, at which the angle contained between the normal and the distance from the center, is a maximum.

7. The place of the earth, when a star's aberration in declination $= 0$, being d_0; and A_0 (lying to the eastward of the point of syzygy) being the place of the aberratic point, when its aberration in right ascension $= 0$; required the position of d_0 with respect to A_0.

8. Solve the following equation $x^3-9x+28=0$ by a process similar to that employed by Cardan.

9. If a small pencil of parallel rays fall upon a concave spherical surface, and every ray be reflected, the density of the incident pencil is to the density (supposed uniform) of rays in the least circle of aberration, as the area of the circle, whose diameter is the versed sine of the aperture, to the whole surface of the sphere, very nearly; required a proof.

10. Find the n roots of unity, and shew how the quadratics combine, when n is an odd number.

11. The weight of a given globe being inconsiderable, when compared with the weight of an equal bulk of fluid, prove that, in its ascent, the velocity is uniform, and equal to the velocity by gravity through $\frac{4}{3}$ds of its diameter.

12. Investigate an expression for the pressure on the axis of a mechanical power in motion, and apply it in the case of the single moveable pulley.

13. Sum the following series:

$$\frac{11}{1.2.3.4}+\frac{17}{4.5.6.7}+\frac{23}{7.8.9.10}+\text{\&c. to } n \text{ terms.}$$

$$\frac{5}{1.2.3}.\frac{1}{2^2}+\frac{6}{2.3.4}.\frac{1}{2^3}+\text{\&c. to } n \text{ terms.}$$

$$\frac{1}{1.2}-\frac{1}{4.5}+\frac{1}{7.8}-\text{\&c. in infinitum.}$$

14. Explain D'Alembert's *principle*; and apply it to find the accelerating force on a body drawn up

an inclined plane, by the action of a power parallel to the plane.

15. In any square number, 4 is the only digit which can occupy both the units and tens places.

16. To find the whole number of equal and regular figures, which may be described upon the surface of a sphere so as exactly to cover it.

17. BM is a chord of a circle, whose center is C, and CEF any radius cutting BM in the point E; at every point E, EP is erected perpendicular to BM and equal to EF; required the locus of P.

18. In the catenary, the horizontal tension is the same at every point, to determine its actual value.

19. Find the following fluents:

$$\int \frac{x\dot{x}}{(1+x)^3 . (1+x+x^2)^{\frac{1}{2}}}, \quad \int b . x^{\frac{1}{2}}\dot{x} . \frac{\sqrt{(2a-x)}}{(a-x)^2},$$

$$\int a z^2 . y^{n-2}\dot{y}, \text{ if } \dot{z} = (b+cy^n)^m . \dot{y}.$$

20. A body, acted upon by gravity, is projected horizontally, with a given velocity, along the interior surface of a cylinder; required to trace its path upon the surface of the cylinder.

21. To calculate the probability of throwing two assigned numbers, A and B, with m dice, in n throws.

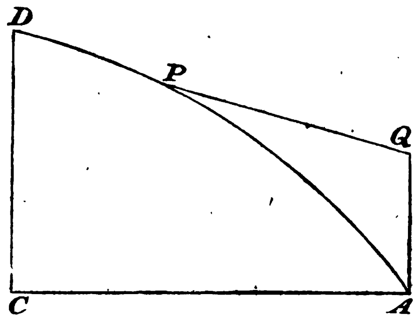
22. Solve the following fluxional equations:

$$(x-y) . \dot{x}\dot{y} = y . \dot{x}^2 + (a-x) . \dot{y}^2,$$

and $\dfrac{\ddot{x}}{\dot{z}^2} + x + \cos. mz = 0.$

23. If an equilateral polygon of 2^n sides be inscribed in a circle, whose rad.$=1$, the value of each side is $\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2\ \&c.}}}}$ where the numeral 2 is repeated $(n-1)$ times.

24. *ADC* is a common parabola, *AQ* a tangent at the vertex, and *PQ* a tangent at *P*, meeting it in *Q*; to determine the point *P*, such, that the resistance



on the solid, generated by the revolution of *DPQA* about *CA*, when moving in the direction of its axis, may be the least possible.

1817.

Monday Morning.—Mr. Peacock.

Monday, January 13, 1817.

FIRST AND SECOND CLASSES.

1. What decimal of £1. is 3*s*. $3\frac{3}{4}$*d*. ?

2. Find the integer values of x and y, which satisfy the equation $13x + 14y = 200$.

3. Prove that

$$\theta = \tan.\theta - \frac{1}{3}\tan.\theta^3 + \frac{1}{5}\tan.\theta^5 - \&c.$$

4. Find the integral of $\dfrac{dx}{1+x^3}$.

6. Explain what is meant by the *particular solutions* of differential equations. Give an instance in the equation

$$ydx - xdy = n\sqrt{\{dx^2 + dy^2\}}.$$

6. Find expressions for the range and time of flight of a body projected from a given point *above* a given plane.

7. Two vessels filled with air of different densities communicate by a tube. Find the velocity with which the air will rush into the vessel containing the rarer air.

8. Explain the causes of the following lunar inequalities : (1.) The evection.
(2.) The variation.
(3.) The annual equation.

9. The aberration which arises from the spherical surfaces of lenses is very small, compared with that which is caused by the unequal refrangibility of light.

10. Prove that in the calculus of variations,

$$\delta dv = d\delta v,$$

v being a function of x.

Monday Afternoon.—Mr. Peacock.

FIFTH AND SIXTH CLASSES.

1. Find the quotient of 75.04 divided by 3.02101 to three places of decimals.

2. Find the amount of £70. in three years, at $3\frac{1}{2}$ per cent. allowing simple interest.

3. Shew that

$$\{\sqrt{(a+b\sqrt{-1})}+\sqrt{(a-b\sqrt{-1})}\}^2 = 2a+2\sqrt{\{a^2+b^2\}}.$$

4. The number N is divisible by 7, if $a_0+a_1.3+a_2.3^2+a_3.3^3+$ &c. $+a_n.3^n$, be divisible by 7; a_0, a_1, a_2, &c. being the digits of the number, reckoning from the place of units.

5. The middle term of the expansion of $(1+x)^n$, when n is even, is

$$=2^{\frac{n}{2}}.\frac{1.3.5\ldots.(n-1)}{1.2.3\ldots\left(\frac{n}{2}\right)}.x^{\frac{n}{2}}$$

6. Explain the method of constructing a table of sines.

7. Solve the equation

$$x^4+px^3+qx^2+px+1=0.$$

8. There are at least as many impossible roots in the original equation as in the equation of limits.

9. Explain what is meant by the modulus of a system of logarithms, and shew how it is determined.

10. If $\pi=3.14159$, $t=$time of oscillation, and l the length of the pendulum, then

$$t=\pi\sqrt{\frac{l}{2mf}}.$$

11. Explain the method of determining the right ascensions of the fixed stars.

12. Enumerate and explain the phænomena exhibited by the moon in the course of a month.

13. Explain the method of determining the specific gravities of bodies.

14. A given rectilinear object is placed at a given point in the axis of a concave mirror. Required the nature and position of its image.

15. Find the fluxion of sin. x.

16. Find the values of x, which make the function $x^3-x^2-8x+12$ a *maximum* or a *minimum*.

17. Expand $(1+x)^{\frac{1}{2}}$ by means of Maclaurin's theorem.

Monday Afternoon.—Mr. White.

THIRD AND FOURTH CLASSES.

1. A sum P is due at the end of m years; find the difference between its amount at the end of $(m+n)$ years, and the amount of its present value at the end of $(m+n)$ years, at simple interest.

2. It is required to find two harmonic means between 3 and 12.

3. If two circles intersect each other in A, B; any two parallel lines CD, EF, drawn through A, B, respectively, and cutting the circles in C, D; E, F; are equal. Required a proof.

4. An object is placed before a concave spherical

reflector. Required its position, when the image is inverted, and equal to twice the object.

5. It is required to find the principal focus of a concavo-convex lens, of a rarer medium, whose thickness is inconsiderable.

6. A body falls down a given length of an inclined plane, and impinging upon the horizontal plane moves along it; required the elevation of the plane, when the time of moving upon the horizontal plane, over a space equal to the height of the plane, is equal to the time down the plane.

7. Prove that when a body oscillates in a cycloid, the whole force which stretches the string, varies as the length of that part of it, which is not in contact with the upper cycloid.

8. The length of the shadow of an upright rod at noon on the shortest day : its length at noon on the longest day :: $n : 1$. Prove that the sine of twice the latitude : the sine of twice the obliquity :: $(n+1) : (n-1)$.

9. A hemispherical vessel standing upon its base is filled with fluid; compare the pressures perpendicular to its plane and curved surfaces.

10. Compare the times of emptying a vessel in the form of a parabolic frustum, by a small orifice in its base, when it is placed with the vertex of the parabola downwards, and when it is placed with the vertex upwards.

11. Compare the time of descent in a given logarithmic spiral, to the center S, from a given point P, with the periodic time in a circle, at the distance SP.

12. If a body fall down the radius of a circle, F varying as (dist.)^3, and ascend on the other side through radius by a repulsive force; shew that it will acquire the velocity of revolution in the circle.

13. In any spherical triangle whose sides are a, b, c, and opposite angles A, B, C; if $b=c$, shew that

$$\sin. b = \frac{\sin. \frac{a}{2}}{\sin. \frac{A}{2}}, \text{ and } \sin. B = \frac{\cos. \frac{A}{2}}{\cos. \frac{a}{2}}.$$

14. Sum the series

$$\frac{3}{1.2.2} + \frac{4}{2.3.2^2} + \frac{5}{3.4.2^3} + \&c. \text{ to } n \text{ terms},$$

$$\text{and } \frac{1}{1.5} + \frac{1}{3.7} + \frac{1}{5.9} + \&c. \text{ to } n \text{ terms}.$$

15. Find the fluents of

$$\frac{p\dot{x}}{\sqrt{(x)}.\sqrt{(a-bx)}}, \text{ and } \frac{\dot{x}}{x\sqrt{(1+\sqrt{x})}}.$$

16. Find the centre of gyration of the plane of a semicircle, revolving about its diameter.

Monday Evening.—Mr. WHITE.

1. The present value of a freehold estate of £100. *per annum*, subject to the payment of a certain sum (A) at the end of every two years, is £1000. allowing 5 *per cent.* compound interest. Required to determine the sum A.

2. $1^3=1$, $2^3=3+5$, $3^3=7+9+11$, $4^3=13+15+17+19$, &c.=&c. Prove the formula for n^3.

3. Two equal hard bodies are projected at the same instant towards each other, from the two extremities of a vertical line, each with the velocity which would be acquired by falling down it. Required the interval of time, between their impact and their arrival at the lower extremity of the line.

4. A hemispherical vessel, of given weight, floats upon a fluid, with one third of its axis below the surface; required the weight which must be put into it, so that it may float with two-thirds of its axis below the surface.

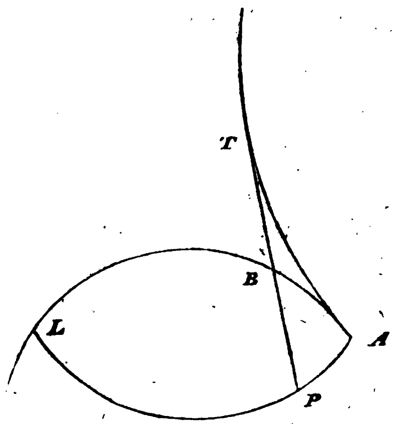
5. A ray of light passes through a prism of a denser medium, and the ray within makes two acute angles with the sides of the prism; if I, i, be the angles which the incident and emergent rays make with the perpendiculars to the surfaces, and R, r, the angles which the ray within makes with the same perpendiculars, prove that when the deviation is a minimum, $I=i$, and $R=r$.

6. If the moon and sun be upon the meridian at the same instant, and A, a, be the increases of their right-ascensions (supposed uniform) in one solar day; A, a, being reckoned in time at 15^0 to one hour; shew that the exact interval between their next following transits $= \dfrac{(A-a)}{24-(A-a)} \times 24$ hours, solar time.

7. Prove that by means of the series of weights 1, 2, 4, 8, 16, &c. any weight not exceeding the sum of the weights, can be weighed.

8. Prove that a circular arc of given radius, will oscillate through a given angle, in its own plane, about its middle point, in the same time, whatever be its length.

9. When a body oscillates in a hypocycloid, as in



the tenth section; if TBP be any position of the

string, shew that the time of describing AP : the time of describing PL :: the arc AB : the arc BL.

10. Prove that the roots of the equation, $x^m+1=0$, when m is an odd number, are, $\frac{1}{r^{m-2}}$........ $\frac{1}{r^3}$, $\frac{1}{r}$, r, r^3,.......r^{m-2}, r^m, where $r = \left(\cos.\frac{\pi}{m} + \sqrt{(-1)}\sin.\frac{\pi}{m}\right)$ and $r^m = -1$.

11. Prove that a regular octahedron inscribed in a sphere : the cube of the radius :: 4 : 3.

12. A body is projected, at a given distance r, at an angle of 30^0, with the velocity acquired from infinity. Find the time elapsed when the body is at the distance $\frac{r}{2}$ from the center; supposing the force to vary as $\frac{1}{(\text{dist.})^4}$, and to be equal to twice the force of gravity at the point of projection. (Newt. Sect. 8.)

13. The sides of a spherical triangle are a, b, c; and the opposite angles A, B, C; if A and C be invariable, and b be increased by a small quantity, shew that a will be increased or diminished, according as c is less or greater than a quadrant.

14. The mean motion of the nodes of the fourth satellite of Jupiter, caused by the disturbing action of the third, ought, according to the principles of the eleventh section, to be regressive; whilst this

regression takes place, can the node of the orbit of the fourth satellite be progressive upon Jupiter's orbit?

15. When rays diverge, from a point beyond it's principal focus, upon a double convex lens of a denser medium; if q' be the distance of the focus of refracted rays from the second surface, the thickness (t) being small; and q that distance when t is neglected; shew that $\frac{1}{q'} = \frac{1}{q} + \frac{(nd-r)^2}{(1+n)\,d^2\,r^2}\cdot t$, nearly; where r is the radius of the first surface, d the distance of the focus of incidence from it, and $1+n : 1 :: \sin. I : \sin. R$.

16. A ball, whose elasticity : perfect elasticity :: $n : 1$, is projected obliquely upwards, from a point in the horizontal plane, upon which it impinges and rebounds continually; prove that the ranges and times of flight in the successive parabolas described, form geometric progressions; and find their sum.

17. If the resistance vary as (vel.)^3, and a body fall by the action of a constant force; find the time in which it will acquire a given velocity.

18. Find the fluent of $\frac{\dot{x}}{(a^2+x^2)^{\frac{2r+1}{2}}}$, and in the fluent of $\frac{x^m\dot{x}}{x^n - px^{(n-1)} + qx^{(n-2)} - \&c.}$, m being greater than n, shew that the coefficients of all the terms which involve higher powers of x than the $(m-n+1)^{\text{th}}$ will vanish.

19. Sum the following series,

$$\frac{1}{1.3.3} - \frac{2}{3.5.3^2} + \frac{3}{5.7.3^3} - \&c. \text{ ad infinitum.}$$

also, $\frac{2}{1.3.3} + \frac{3}{3.5.3^2} + \frac{4}{5.7.3^3} + \&c.$ to n terms by the method of increments.

20. An isosceles right-angled triangle is immersed in a fluid, having one of its sides coincident with the surface; find the distance of the center of pressure from the side immersed.

21. (Sect. XI. Prop. 66.) Cor. 14. If ST and the absolute force of S be changed, the periodic linear errors of $P \propto \frac{1}{(\text{per}^{\text{c}}.\text{ time of } T)^2}$. Cor. 15. If ST, PT, be changed in the same proportion, and also the absolute forces of S and T be changed in the same proportion, the periodic linear errors of P vary as PT. Required proof: and hence to compare the periodic linear errors of P in different systems of S, T, P, where the form and inclination only of the orbits remain the same.

22. If out of 86 persons born, one dies at the end of every year; and m, n, be the complements to 86 of the ages of two individuals A, B, m being less than n, prove that the probability of A's surviving $B = \frac{m-1}{2n}$.

23. In a system of two pulleys, where each string is attached to the weight, P draws up W; find the

accelerating force on P, the tensions of the strings, and the pressures upon the centers of the pulleys; taking into consideration the weight and inertia of the pulleys.

Tuesday Morning.—Mr. White.

FIRST AND SECOND CLASSES.

1. The logarithm of 37852 is 4.5787767; the logarithm of 37853 is 4.5787882; required the number, corresponding to the logarithm 6.5787836.

2. If the sides of a spherical triangle, AB, AC, be produced to b, c, so that Bb, Cc, shall be the semi-supplements of AB, AC, respectively; prove that the arc bc will subtend an angle at the center of the sphere, equal to the angle between the chords of AB, AC.

3. If the radii, of the tube, and of the basin, of a barometer, be 1 and 3; and the index shews, at sight, the height of the mercury in the tube, above that in the basin; prove that the inch upon the scale : a real inch :: 8 : 9, the thickness of the tube being neglected.

4. The altitudes of a circum-polar star are observed at two instants, when it has the same azimuth, before it passes the meridian; and also the time between those instants; from these data, determine the latitude of the place.

5. Having given the distance at which a short-sighted person can see distinctly, it is required, to find the distance between a given object-glass, and given eye-glass, in an astronomical telescope, when adapted to such an eye, and to distant objects.

6. The periodic times of the first and second satellites of Jupiter, are, ($1^d.\ 18^h.\ 27^m.\ 33^s.$) and ($3^d.\ 13^h.\ 13^m.\ 42^s.$) If a, a', be their mean distances, prove that, $a : a' :: 1 : 2^{\frac{2}{3}} \times \left(1 + \frac{2}{3} \times \frac{1}{274}\right)$ nearly.

7. If the $\sqrt[3]{\tan.\left(45 - \frac{z}{2}\right)} = \tan.\ u.$ prove that, $\sqrt[3]{(\tan.\ z + \sec.\ z)} + \sqrt[3]{(\tan.\ z - \sec.\ z)} = 2 \cot.\ 2u.$

8. Determine the weights which must be selected out of the series, 1, 2, 4, 8, &c. pounds, in order to weigh 1317 pounds.

9. If a body be projected obliquely upwards, shew that the square of its velocity, will always be equal to the square of the velocity of projection, diminished by the square of the velocity which it would acquire by falling down its perpendicular height, above the horizontal plane passing through the point of projection.

10. A body describes a circle, the center of force being in the circumference; another body describes an equal circle, the center of force being in the center of the circle, and the absolute force being

one-fourth of its former value. Compare the times in which the circles are described.

11. Prove, that $(a+b)^n = a^n + n.a^n.\frac{b}{(a+b)}$ $+ n.\frac{n+1}{2}.a^n.\frac{b^2}{(a+b)^2} + n.\frac{n+1}{2}.\frac{n+2}{3}.a^n.\frac{b^3}{(a+b)^3}$ + &c. and by this theorem, sum the series, $\frac{1}{3}+\frac{3}{2}.\frac{1}{3^2}+\frac{3.4}{2.3}.\frac{1}{3^3}+\frac{3.4.5}{2.3.4}.\frac{1}{3^4}+$ &c. ad infinitum.

12. Upon one side of the given straight line *AB* describe a semicircle, and upon the other side an equilateral triangle *ADB*; if a solid be generated by the revolution of this figure about *DC*, *C* being the center of the semicircle; prove that it will rest upon the horizontal plane, upon any point of it's spherical surface.

Tuesday Afternoon.—Mr. White.

FIFTH AND SIXTH CLASSES.

1. It is required to express 23°. 27′. 53″. in hours, minutes, and seconds.

2. Find the discount upon £125. 10*s*. 0*d*. payable at the end of three years, at 4½ *per cent*. simple interest.

3. It is required to determine the point *C* in the semicircle *ACB*, such that the three sides of the triangle *ACB* shall be in geometrical progression.

4. Two bodies 1, 2, moving with velocities 1, 2, whose elasticity : perfect elasticity :: 1 : 2, impinge upon each other, making the angles of 30^{0}, and 90^{0}, respectively, with the plane touching them at the point of contact. Required the directions in which they will move, and their velocities after impact.

5. A body is projected down an inclined plane, with the velocity acquired in falling down its height, and describes the length of the plane in the time of falling down its height. Required the elevation of the plane.

6. In a quadrantal triangle, the angle opposite the quadrant, and one of the other angles, are given; find the remaining angle.

7. Prove that the illumined phase of Mars is the least, when he is in quadrature.

8. If an object be viewed through a glass plate of given thickness, determine how much the apparent distance is less than the true.

9. It is required to determine the brightest part of the visible area in Galileo's telescope.

10. A circle and its inscribed hexagon, move with equal velocities, in directions inclined at angles of 30^{0} and 60^{0}, respectively, to their planes. Compare the resistances perpendicular to their motions.

11. Sum the following series to n terms,

$$r - \frac{r^2}{2} + \frac{r^3}{4} - \&c.$$

and $1.2 + 2.5 + 3.8 + 4.11 + \&c.$

12. Find the fluents of

$$\frac{p\dot{x}}{a - bx^2}, \text{ and } \frac{x^3\dot{x}}{\sqrt{(a-x)}}.$$

13. Having given the ratio of the periodic times in two circles, described about different centers of force situated in their centers, and also the ratio of the radii, it is required to find the ratio of the absolute forces.

14. Determine the angle between the apsides in an orbit nearly circular, the force being constant; taking an ellipse about the center for the revolving orbit.

Tuesday Afternoon.—Mr. Peacock.

THIRD AND FOURTH CLASSES.

1. If $\frac{p_0}{q_0}$ and $\frac{p_1}{q_1}$ be two consecutive terms in a series of fractions converging towards $\frac{a}{b}$; then

$$p_0 q_1 - p_1 q_0 = \pm 1.$$

2. Explain what is meant by the *conjugate points* of curve lines.

3. If $u = f\{x, y\}$, shew that

$$\frac{d^2u}{dx\,dy} = \frac{du^2}{dy\,dx}.$$

4. Find the integral or fluent of

$$\frac{dx}{x^3 - 7x^2 + 12x}.$$

5. If a body be projected perpendicularly upwards with a velocity (a), its height (x), at the end of the time (t), is determined from the equation

$$(a - 2mt)^2 = 4m\left(\frac{a^2}{4m} - x\right).$$

6. Enumerate the different practical methods of determining the latitude of a ship at sea.

7. Explain the method of measuring altitudes, by means of the barometer and thermometer.

8. A given rectilinear object is placed before a spherical reflector of given radius. Find the equation to the conic section which is its image.

9. Find an expression for the whole time of descent of a body from a distance (a) to the center of force, when the force varies inversely as the square of the distance.

10. Mention some of the problems, upon which the trisection of an angle, by common geometry, may be made to depend.

Tuesday Evening.—Mr. Peacock.

1. Demonstrate the rule for the extraction of the square root in numbers.

2. Every prime number of the form $4n+1$ is the sum of two squares.

3. Approximate to the value of x in the equation

$$x^3-2x-5=0,$$

and explain the defects of the methods of approximation, as given by Newton and Ralphson.

4. Prove that

$$\Delta^n u_x = u_{x+n} - \frac{n}{1}\,.\,u_{x+n-1} + \frac{n(n-1)}{1\,.\,2}\,u_{x+n-2} - \&c.$$

5. Integrate the differentials

$$\frac{dx}{x^5\sqrt{\{1+x^2\}}},\quad \frac{a^x\,dx}{x^2},\text{ and } dx\cos.^2 x\sin.^3 x.$$

6. Integrate the differences or increments

$$x^3 \text{ and } e^x\cos.\,x\,\theta.$$

7. Integrate the different equations

(1.) $\dfrac{d^2 y}{d\,x^2} = \dfrac{m}{(a-y)^2}$.

(2.) $d^2 y + Aydx^2 = Xdx^2$, where X is a function of x.

(3.) $\dfrac{dz}{dx} - \dfrac{y}{x}\cdot\dfrac{dz}{dy} = -\dfrac{y^2}{x^2}$.

8. Integrate the equation of differences,

$$u_{x+2} - Au_{x+1} + Bu_x = 0.$$

9. Given the length of the curve; required its nature when its centre of gravity is most remote from the axis.

10. If two lines intersect each other within a parabola, the ratio of the rectangles contained by their respective segments will be the same with the ratio of the rectangles made by the segments of any other two lines which intersect each other, and which are respectively parallel to the former.

11. Apply D'Alembert's principle to the determination of the distance of the centres of oscillation and suspension in a compound pendulum.

12. A triangular prism being immersed in a fluid of greater specific gravity than itself, it is required to determine the different positions in which it will rest in equilibrium.

13. A machine, driven by the impulse of a stream, produces the greatest effect when the wheel moves with one-third of the velocity of the water.

14. At a place whose latitude is $48^\circ. 50'. 14''$, the meridian altitude of the sun's upper limb was observed to be $62^\circ. 29'. 56''$; it is required to determine the sun's declination, the refraction being $29''$, the sun's parallax and apparent diameter of their mean values, and the sine of $27^\circ. 30'. 4'' = .4617$.

15. Explain the method of correcting an error in the longitude of a place, by means of the occultation of a given fixed star by the moon.

16. If r be the radius of an isosceles lens, whose focal length is equal to that of a lens whose radii are r_1 and r_2; then

$$\frac{1}{r} = \frac{1}{2}\left\{\frac{1}{r_1} + \frac{1}{r_2}\right\}.$$

17. If D be the length of a degree of the meridian at a point whose latitude is λ, Δ the length of a degree of a curve perpendicular to the meridian at that point, a the axis major of the meridian, and e the difference of the semi-axes; then

$$\frac{e}{a} = \frac{\Delta - D}{2\,\Delta \cos.^2 \lambda} \text{ (nearly).}$$

18. The moon is retained in her orbit by the force of gravity. Newton. Lib. III. Prop. 4.

19. The sum of the sides of a right-angled triangle remaining the same, required the nature of the curve to which the hypothenuse is always a tangent.

20. Explain the method of drawing a normal to a given curve surface.

21. Give an account of the controversy between the followers of Newton and Leibnitz, concerning the measure of motion; and reconcile the experiments and results to which the latter appealed, with the measure assumed by the former.

22. If two chords of a circle intersect each other at right angles, the sum of the squares described upon the four segments is equal to the square described upon the diameter.

23. Give some account of the analysis of the ancient geometers. Exemplify it in the solution of the following problem: "To bisect a triangle by a straight line drawn through a given point in one of its sides."

1818.

Monday Morning —Mr. French.

Monday, January 20, 1818.

FIRST AND SECOND CLASSES.

1. The present value of an annuity, to continue for a term of years at a given rate of compound interest, $= m \times$ the present value of the same annuity, to be paid only during the latter half of the same term; required to find when the annuity will cease.

2. To determine the numerical value of the arc A which will *satisfy* the following equation:

$$\sin. B + \sin. (A-B) + \sin. (2A+B) = \sin. (A+B) + \sin. (2A-B).$$

3. Prove that the sum of all the coefficients of a binomial raised to the $(2n)^{\text{th}}$ power : the coefficient of its middle term :: $2.4.6$, &c. to n factors : $1.3.5.$ &c. to n factors.

4. A body is suspended from a given point in the horizontal plane, by a string of known length, which is thrust out of its vertical position by a rod

(supposed without weight) acting, from a given point in the plane, against the body; shew that the tension of the string varies inversely as the tangent of the inclination of the rod to the horizon.

5. Two equal hollow paraboloids have a common axis, which is vertical, and such a quantity of water is poured in between them, as just to touch the lowest point of the inner figure; demonstrate that the surface of the water will be a tangent plane to this figure, in any position of the common axis.

6. In Gregory's telescope, the focal length of the larger reflector, the position and focal length of the eye-glass, and the distance between the two images of a remote object being given; required to find the position and focal length of the smaller reflector, which will cause the telescope to magnify the object in any proposed ratio.

7. Having given $nt = u - e.\sin. u$; required the first four terms of the series expressing u in terms of nt.

8. A spherical body descends in a fluid by gravity; to determine the quantity of the resistance, when the body has described a given space.

9. The force varying inversely as $(\text{dist.})^5$ and the velocity being that which would be acquired from infinity, a body is projected from an apse; compare the time of its descent to the center, with the periodic time in a circle, whose radius = half the apsidal distance.

10. To find the fluent of $\dfrac{\dot{x}\sqrt{(1-x^2)}}{(1+x)^2}$.

11. Sum the following series:

$$\frac{1}{1.1}+\frac{1}{3.5}+\frac{1}{5.9}+\text{\&c. in inf.}$$

12. A body describes a parabola about the focus, and at the same time the figure moves uniformly in a direction *perpendicular* to its axis, which continues parallel to itself; to determine the path described by the body in fixed space.

Monday Afternoon.—Mr. French.

FIFTH AND SIXTH CLASSES.

1. $\dfrac{x}{x+2}-\dfrac{x-9}{3x-20}=\dfrac{9}{13}$. Required the values of x.

2. Required the ratio which is one half of the ratio ($\sqrt{32}$: 25.)

3. The sum of an arithmetic series is 5, the first term 11, and the common difference -5; find the number of terms.

4. To determine the value of tan. 30^0 to two places of decimals. (rad. = 10000.)

5. P being any point in an ellipse, whose semi-axis major is AC; prove, that, if the normal (PG)

be produced to meet the conjugate diameter in F and the minor axis in V,

$$PF.PV=(AC)^2.$$

6. Two circles touch each other internally, and the area of the *lune* cut out of the larger is equal to twice the area of the smaller circle. Required the ratio of the diameters of these circles.

7. A body is projected perpendicularly upward with a velocity of 64 feet per second; find the time of ascent through 63 feet.

8. The length of the gage of a condenser is 12 inches, and the space occupied by the air in it, after two descents of the sucker, is half its whole length; to determine the space which the air will occupy after the third descent of the sucker.

9. Having given the focus of incidence of a pencil of rays which passes nearly perpendicularly through the sides of a prism, and sin. I : sin. R :: n : 1 out of the ambient medium into the prism; required (n being a proper fraction) to find the focus of emergent rays.

10. If a star be situated nearer to the pole of the ecliptic than to that of the equinoctial, shew that its right ascension exceeds 180^0.

11. A body is revolving in a given circle about its center, if the absolute central force be increased in a given ratio, what change must be made in the

velocity of the body, that it may still describe the same circle?

12. Demonstrate, as Newton has done (Cor. 2. Prop. 10.), that the periodic times in all ellipses about the same center are equal.

13. Assuming the velocity to vary as $\frac{\sqrt{(a-x)}}{\sqrt{(x)}}$, a being the initial distance and x the variable distance of the body from the center of force; to determine the law of the centripetal force.

14. One root of the equation $(x^3-11x^2+37x-35=0)$ is $3+\sqrt{2}$; required the remaining roots.

15. Required the fluxion of $\frac{y\sqrt{(a^2-y^2)}}{a^3-y^3}$.

16. Find the fluent of $\frac{az^2\dot{z}}{e-fz}$.

Monday Afternoon.—Mr. Fallows.

THIRD AND FOURTH CLASSES.

1. What part of half a crown is equal to $\frac{1}{7}$ of 1*s*. 5½*d*.

2. Of all triangles under a given perimeter and a determinate side, shew that to be the greatest in which the two indeterminate sides are equal.

3. If the p^{th} and q^{th} terms of an arithmetical progression be P and Q, find the sum of n terms of the series.

4. Transform the cubic equation $x^3+px^2+qx+r=0$, whose roots are a, b, c, into another whose roots are

$$\left(\frac{1}{a^2}+\frac{1}{b^2}\right), \quad \left(\frac{1}{a^2}+\frac{1}{c^2}\right), \quad \left(\frac{1}{b^2}+\frac{1}{c^2}\right).$$

5. A ship sails directly north at the rate of (a) miles an hour, and the velocity of the wind is (b) miles an hour; find the direction of the wind so that the vane may point due west.

6. Find the quantity of water discharged from a small given orifice in the side or bottom of a vessel in a given time; the vessel being kept constantly full.

7. Having given the radius of an arc of any colour in the secondary rainbow, find the ratio of the sine of incidence to the sine of refraction when rays of that colour pass out of air into water.

8. If a body revolve in an ellipse (whose major and minor axes are given) with the force tending to its focus, and the time of revolution be given; find the actual velocity of the body at any given point in its orbit.

9. If the hyp. log. $\dfrac{\sqrt{(a^2+x^2)}+a}{\sqrt{(a^2+x^2)}-a}=b$, find x.

10. Find the surface of the solid generated by the revolution of a common cycloid about its axis.

11. Explain why the effect of aberration on a star not situated in the solstitial colure, at six o'clock, either evening or morning, is partly in declination and part in right ascension.

12. A luminous point is placed in the axis of a glass lens, which is *plane* on one side and *curved* on the other; find the nature of the curved surface so that rays diverging from the luminous point, may after passing through the lens, be refracted accurately to another given point.

13. The right ascension and declination of a star being given, as also the time of the year when it rises with the sun; find the latitude of the place.

14. The increment of the hyp. log. $(x) =$

$$2\left\{\left(\frac{\dot{x}}{2x+\dot{x}}\right)+\frac{1}{3}\left(\frac{\dot{x}}{2x+\dot{x}}\right)^3+\frac{1}{5}\left(\frac{\dot{x}}{2x+\dot{x}}\right)^5+\&c.\right\}$$

15. Find the following fluents:

$\int \frac{x^2\dot{x}}{(a^2+x^2)^2}$; $\int \frac{a^x\dot{x}}{\sqrt{(1-a^{2x})}}$; $\int z\,\dot{x}$, where z is an arc whose tangent $= x$.

Monday Evening.—Mr. Fallows.

1. If £1. £8. £27. &c. be lent at the beginning of the first, second, third, &c. year; find the whole

amount due, at simple interest, at the end of n years, at r rate per cent.

2. Given the four sides of a quadrilateral figure inscribed in a circle; to find its diagonals.

3. A string passing over a fixed pulley is coiled, on each side of it, round two cylinders of equal weight (w), the one being of uniform density, the other collected in the circumference; find the tension of the string when they are at liberty to move; the inertia of the string and pulley not being taken into account.

4. Given the area of a right-angled triangle; to find the curve to which the hypothenuse is always a tangent.

5. At what angle must two plane reflectors be inclined, so that a man standing in a given position, may see his face in *profile* in the image of one of them?

6. The ages of two persons being equal; find the value of an annuity of £1. for their joint lives.

7. A body revolves in an ellipse, the force being in the focus; shew that if an additional velocity be communicated to it in its descent from the higher to the lower apse, the apsides are regressive, and if communicated in its ascent from the lower to the higher, they are progressive.

8. Two barometers whose lengths are a, a' inches, contain b, b' inches of air respectively; if on account

of some change in the weather the former barometer falls one inch, what will be the depression in the latter; supposing a perfect barometer to stand at 30 inches before the depression?

9. Equal altitudes of the sun are taken before and after its passage over the meridian, and the times of observation noted by a chronometer; find its error when the change of declination is taken into account.

10. Find the integral of cos. z; and from thence, sum the series cos. a + cos. $(a+b)$ + cos. $(a+2b)$ ...... + cos. $(a+nb)$.

11. Find the following fluents:

$$\int x^2 \dot{x}.\text{arc. }(\sin.=x), \quad \int \frac{\dot{x}(1+x^2)}{(1-x^2).\sqrt{(1+x^4)}}.$$

12. Find the relation between x and y, in the following equations:

$$x\dot{y} - y\dot{x} - (x^2+1)\,\dot{x} = 0;$$

$$(\dot{x}^2+\dot{y}^2)^{\frac{3}{2}} + 2a^{\frac{1}{2}}\sqrt{(a-x)}\,.\,\dot{x}\ddot{y} = 0.$$

13. If the mean density of the earth (considered as a sphere) be to the density at the surface as 1 : m; find that power of the distance from its center according to which the density of its parts varies.

14. Explain the construction of *Mercator's* chart, and from thence find the distance of two places projected on the chart whose latitudes and longitudes are given.

15. If a prolate spheroid be cut by a plane passing through the focus, the section will be an ellipse having its focus in the focus of the spheroid.

16. Prove that the sum of the terms of *Taylor's* series commencing with any given term, can always be rendered *less* than the term immediately preceding it.

17. If a small pencil of parallel homogeneal rays be refracted into a sphere of water, and emerge parallel; shew that after two refractions and one reflection, the angle contained between the incident and emergent rays is a *maximum*, and after two refractions and two reflections it is a *minimum*.

18. A circle has the greatest triangle inscribed in it, a circle is inscribed in the triangle, which has the greatest triangle inscribed in it, and so on; find the sum of all the circles and triangles.

19. If $F \propto \frac{1}{D^3}$ and be attractive; shew that six different kinds of orbits may be described with proper velocities and angles of projection, and only six; and when repulsive, only one.

20. A paraboloid rests upon an horizontal plane with its axis vertical and vertex downwards: what must be the length of its axis in order that the equilibrium may be that of *indifference?*

21. If the resistance of the medium vary partly in

the simple and partly in the duplicate ratio of the velocity, and a body urged by the force of gravity ascend or descend in the medium; shew how the spaces described by the body in different times may be compared. *Newton*. Prop. 14. Book II.

22. A rigid prismatic bar of uniform density and given length is placed in the straight line joining two centers of force, whose distance is given, and whose intensities are in the ratio of 2 to 1; find the position of the bar so that it may rest in equilibrium, supposing $F \propto \frac{1}{D}$.

23. The lunar orbit being supposed circular; compare the moon's velocity in quadratures with its velocity at any given place of its orbit, taking into consideration that the earth and moon revolve about their common center of gravity. *Newton*. Prop. 26. Book III.

24. Investigate the following formula for clearing the moon's distance: ver. sin. (D) = ver. sin. $(A-B) + \frac{\cos. A . \cos. B}{\cos. a . \cos. b}$ {ver. sin. (d) — ver. sin. $(a-b)$} where A, B; a, b, are the true and apparent altitudes; D, d the true and apparent distances.

Tuesday Morning.—Mr. Fallows.

FIRST AND SECOND CLASSES.

1. Find two fractions whose denominators are prime to each other and their sum $\frac{32}{45}$.

2. The area of a trapezium is equal to the product of its two diagonals multiplied by half the sine of the angle formed by their intersection.

3. In the expansion of $\dfrac{a+bx+cx^2}{1-ax-\beta x^2-\gamma x^3}$; find the general term.

4. Given the lengths of two ordinates of the logarithmic curve and the portion of the abscissa intercepted between them; to construct the curve.

5. Find the *position* of the center of gravity of any number of bodies situated in different planes.

6. If a body fall by the action of an uniform force and describe (a) and (b) feet in the m^{th} and n^{th} second respectively, (reckoning from the beginning of the motion); find the space described in the x^{th} second.

7. Two given glass meniscuses of the same diameter and the same focal length being joined together with their convex sides outwards and the included space being filled with water; find the

focal length of the lens, its thickness not being considered.

8. If an oblate spheroid, whose axes are given, be filled with water and placed with its major axis perpendicular to the horizon; find the time of emptying through a small given orifice at the extremity of the vertical axis.

9. Prove, *strictly*, that $\dot{v} = F\dot{t}$, $v\dot{v} = F\dot{x}$, and $\frac{\ddot{x}}{\dot{t}^2} = F$.

10. In a given latitude and longitude, a vertical plane declines (a^0) from the south towards the west; find the place to whose horizon the plane is parallel.

11. If a body fall from rest through a given space AB towards a given center of force C, in t seconds; compare the force at A with gravity, supposing $F \propto \frac{1}{D^3}$.

12. Investigate the nature of the curve, in which lines drawn from a given point perpendicular to the tangent may always be equal.

13. Find the integral of $\frac{x}{a^x}$.

14. If an elastic chord of uniform density, whose length is (L) and weight (W), be stretched in an horizontal position by a given weight (w) and the increment of length be (l); find the length of the

chord when suspended by one of its extremities; the increment of its length being always as the weight which stretches it.

Tuesday Afternoon.—Mr. Fallows.

FIFTH AND SIXTH CLASSES.

1. A person sold goods to the value of £1000, and gained 20 per cent. What was the prime cost?

2. Surd roots of the form $\pm\sqrt{(b)}$ enter equations by pairs.

3. If two triangles are to each other as their bases; prove that they have the same altitude.

4. A body is projected up a plane inclined to the horizon at an angle of 30^0 with a velocity of 20 feet per second, find where it will be at the end of four seconds.

5. If $\sin.\ (A-B) = \frac{1}{2}$ rad. and $\sin.\ (A-B) = \cos.\ (A+B)$, find A and B.

6. Find how far a body will fall from rest, while a pendulum whose length is 20 inches makes 10 vibrations.

7. Define logarithms, and shew from the definition that $\log.\ (ab) = \log.\ a + \log.\ b$; $\log.\ \left(\frac{a}{b}\right) = \log.\ a - \log.\ b$; $\log.\ a^n = n \log.\ a$.

8. A hollow globe is filled with fluid; compare the internal pressure with the weight of the fluid.

9. In the magic lantern, prove that no image will be formed upon the screen, unless the distance between the lantern and the screen be greater than four times the focal length of the lens.

10. The sun is at the same altitude at equal intervals of time before and after its passage over the meridian, supposing no change in declination to have taken place during the interval.

11. If $F \propto \frac{1}{D^2}$; a body revolving in a circle at a given distance from the center will by its motion at any point turned upwards ascend to twice its distance from the center.

12. Find the following fluents:

$$\int \frac{a\dot{x}}{b+\frac{c}{x}}, \quad \int \frac{x\dot{x}}{(1+x^4)^{\frac{3}{4}}}.$$

13. The circumference of a circle to its diameter is nearly in the ratio of 22 to 7.

14. Inscribe the greatest parallelepiped in a sphere.

15. Every inscribed triangle formed by any tangent and the two intercepted parts of the asymptotes of a hyperbola, is equal to a given area.

16. Find the radius of curvature at the vertex of a common parabola.

17. If a body revolve in a logarithmic spiral, find the law of centripetal force tending to the pole of the spiral.

Tuesday Afternoon.—Mr. FRENCH.

THIRD AND FOURTH CLASSES.

1. If A, B, C, be the three angles of a plane triangle, having given $\cos. B = \frac{1}{2} . \frac{\sin. A}{\sin. C}$; prove the triangle to be isosceles.

2. Prove that the arc $\frac{60^0}{2^{12}} = \left(2^2 - \frac{1}{2^2}\right)^3$ seconds $= 52''. 44'''. 3^{iv}. 45^{v}$.

3. If a body be projected perpendicularly upward, the time of its ascent through any space is determined from the quadratic equation ($mT^2 - V.T + S = 0$); shew that the least root is that which answers the conditions of the problem.

4. If a given pendulum be made to oscillate in a cycloid and in a circle, its greatest velocity in the cycloid : its greatest velocity in the circle :: the cycloidal arc described in its descent : the chord of the circular arc described.

5. A solid of revolution, whose axis is perpendicular to the horizon, empties itself by a small given orifice; required its nature, when the velocity of the descending surface varies inversely as the ordinate of the generating figure.

6. An eye being placed so as just to see the lowest point of an hemispherical vessel, when empty; it is required to determine the perpendicular depth of that point of its inner surface nearest to the eye, which is brought into view when the vessel is filled with water.

7. To a spectator in the northern hemisphere, the sun, whose declination = 15°, rises just two hours before noon; prove that tan. latitude of the place of observation $= \frac{1}{2}\sqrt{3}\sqrt{\frac{(1+\frac{1}{2}\sqrt{3})}{(1-\frac{1}{2}\sqrt{3})}}$. (rad. = 1).

8. A cylinder, whose weight = 133.6 lbs. and rad. = 10, revolves about its horizontal axis; to determine the time in which a weight of 20lbs. acting by means of a string at the circumference of the cylinder, will generate a velocity of 1 foot per second at a distance = 1 from the axis. (m = 16 feet.)

9. If bodies move in a logarithmic spiral from different points to its pole, shew that the times of their motion are as the squares of the spaces which they respectively describe.

10. According to what power of the distance must the force vary, that the areas, *dato tempore*, in all

circles uniformly described about the center of force, may be equal?

11. If the point A (Prop. 41. Sect. viii.) be removed to an infinite distance from the center of force, shew from Newton's construction (Cor. 3.), that the hyperbolic spiral will become a circle.

12. The length of the catenary $= a(e^{\frac{D}{a}} - e^{-\frac{D}{a}})$, D being its greatest ordinate and a the lateral tension. Required a proof.

13. $y = \frac{x \cdot \sqrt{(1-x^2)}}{\sqrt{(1-a^2 x^2)}}$. Required the *maximum* value of y.

Tuesday Evening.—Mr. French.

1. A person transfers £1000. stock from the *five per cents.* to the *three per cents.* when the former are at 110, and the latter at 84; if, at the end of six months, the *five per cents.* have risen to 112, what must then be the price of the *three per cents.* that he may sell out without having gained or lost by the transfer?

2. Having given two distances from the focus of a parabola and the angle between them; to construct the parabola.

3. To determine the greatest straight line which

can be drawn from a given point in the minor axis of an ellipse to its periphery.

4. A ball is projected from a given point in the horizontal plane at an angle of 30^0, and after describing two-thirds of its horizontal range, strikes against a sonorous body; having given the whole interval between the instant of projection, and the instant when the sound reaches the point of projection, to find the initial velocity.

5. The periodic times of four bodies being 24, 22, 20 and 18 days, respectively; in what time, after leaving a conjunction, will they all be again in conjunction, and what number of revolutions will each have performed?

6. If rays fall nearly perpendicularly upon a spherical refracting surfaee of a denser medium converging to a point between the surface and its center, and sin. I : sin. R :: m : n; shew that the greatest distance between the conjugate foci $= \frac{\sqrt{m}-\sqrt{n}}{\sqrt{m}+\sqrt{n}}.r$: (r being the radius of the refracting surface.)

7. The values of an *oz.* of platina, gold and silver being p, g and s respectively, and their specific gravities a, b, c; compare the value of a coin, made of platina and silver, and which equals a guinea in weight and magnitude, with the value of a guinea.

8. Shew that the n^{th} term in the series of hexa-

gonal numbers is the same with the $(2n-1)^{\text{th}}$ term in the series of triagonal numbers.

9. The point C is such that all straight lines drawn from it to two given points A, B, are in a given ratio; prove that the *locus* of C is the circumference of a circle.

10. A small object is placed at such a point in the diameter of a sphere of water as to be distinctly seen, after one reflection and one refraction, by an eye in that diameter produced; compare its visual angle with the visual angle of the same object when placed in the principal focus of the sphere.

11. Find the following fluents:

$$\int \frac{a\dot{x}}{\sqrt{(1-x^4)}}, \text{ when } x=1; \quad \int \frac{(a^2+x^2)^m \cdot \dot{x}}{(\text{hyp. log. } x)^n}.$$

12. A wheel, in 36 revolutions, passes over 29 yards, and in x of these revolutions it describes
yds. feet. inches.
$z + y + 5$; to find the values of x, y and z.

13. To find the place of a body in an elliptic trajectory at any given time. (*Newton*. Vol. I. Sect. 6.)

14. Deduce *Kepler's* law of the equable description of areas about the center of force from the three fluxional equations of motion.

15. Investigate the expression for the precession in right ascension of a star, whose right ascension is greater than 180° and less than 270°.

16. Required the sum of the terms of a binomial $(a+x)^m$, at intervals of n from each other, beginning with the $(p+1)^{\text{th}}$ term.

17. A given hemispherical vessel, whose thickness is t, resting upon its base, is filled with fluid to a depth = half of its inner radius; required the ratio of the specific gravities of the vessel and the fluid, when the vertical pressure of the fluid = the weight of the vessel.

18. To resolve $(a^2 - ab.2\cos.\theta + b^2)^{-\nu}$ into a series of cosines of arcs, the multiples of θ, by means of the formula $2\cos. m\theta = x^m + \frac{1}{x^m}$, and the binomial theorem.

19. A body moves in a logarithmic spiral, the centripetal force varying inversely as $(\text{dist.})^n$, and the resistance as the density of the medium and the square of the velocity jointly; from these *data* determine the law of the density.

20. Sum the following series:

$$\frac{8}{1.3.5.7} + \frac{16}{5.7.9.11} + \frac{24}{9.11.13.15} + \&c.$$

to n terms by increments.

$ax^{a} + (a+b)\,x^{a+\beta} + (a+2b)\,x^{a+2\beta} + \&c.$ to n terms.

21. If seven balls be drawn from a bag containing eleven in all, five of which are white and six black; what is the probability that three white balls will be drawn?

22. Prove that the sum of all the numbers of n places, which can be formed with the n digits a, b, c, &c. : sum of all the numbers of n places which can be formed with the n digits p, q, r, &c. of the same scale :: $a+b+c+$ &c. : $p+q+r+$ &c.

23. In a revolving fluid spheroid of small eccentricity, shew that, if $\sin.^2$ lat. $=\frac{1}{3}$, the distance from the center (CP) = the radius of an equi-capacious sphere, and that the central attraction of P arising from the mutual gravitation of the particles of the spheroid, is equal to its attraction to the same sphere at rest.

24. $ABCDE$ is a pentagonal billiard-table, with unequal but given sides and angles; it is required to find that point in one of its sides, and the direction of impact, such, that an elastic ball may continually describe the same path, striking every side of the table in succession.

1819.

Monday Morning —Mr. Peacock.

Monday, January 18, 1819.

FIRST AND SECOND CLASSES.

1. What number of degrees, minutes and seconds are contained in an arc equal to radius?

2. If from a point without a parallelogram, lines be drawn to the extremities of two adjacent sides and of the diagonal which they include; of the triangles thus formed, that, whose base is the diagonal, is equal to the sum of the other two.

3. If Mx^{n-m} be the first negative term of the equation

$$x^n + px^{n-1} + \dots - Mx^{n-m} - \dots = 0.$$

and if P be the greatest negative coefficient, then $1 + \sqrt[m]{P}$ is greater than the greatest root of the equation.

4. If the inverse ratio of any two consecutive coefficients of the series

$$a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \text{\&c.}$$

be finite, it is always possible to assume x so small,

that any one term of the series may exceed the sum of all those which follow it.

5. In the direct collision of bodies, the velocity of the centre of gravity is the same before and after impact.

6. The bulb of a thermometer is successively plunged into boiling water and melting ice, and the mercury in the tube falls a inches: given the diameter of the tube, and the diminution of bulk due to one degree of temperature, to find the capacity of the bulb.

7. If rays nearly parallel, are incident upon a concave spherical reflector, whose radius is r, and if d and d' be the distances of the foci of incident and reflected rays, then

$$\frac{1}{d} + \frac{1}{d'} = \frac{2}{r}.$$

8. Explain what is meant by the line of *collimation*; and shew by what means any error arising from it, may be compensated in the circular transit instrument with an azimuth motion.

9. Explain the method of finding the longitude, by observing the increase of the moon's right ascension, in the interval of her transit over two meridians.

10. Two lines AP and BP in the same vertical plane, pass through two points A and B situated in the same horizontal line: find the locus of the point

P, so that the time of a body's descending down AP and ascending up BP with the velocity acquired, may be constantly the same.

11. Integrate the differential equation

$$e^x\,dx - \frac{ydy}{e^x} = dy - y\,dx.$$

12. All epicycloids, the radii of whose generating circles bear an assignable numerical ratio to the radii of their bases, are expressible by finite algebraical equations.

13. The cycloid is the curve of quickest descent, between two points which are not in the same vertical line; demonstrate this by the calculus of variations.

Monday Afternoon.—Mr. Peacock.

FIFTH AND SIXTH CLASSES.

1. What is the purchase money of £156. 15*s*. 1*d*. 3 per cent. annuities, at $74\frac{1}{2}$ per cent.?

2. Give the reason why quadratic equations admit of two solutions.

3. Investigate an expression for the number of combinations of n things, taken m and m together.

4. Explain in what case and for what reason,

Cardan's formula for the solution of a cubic equation, does not enable us to determine the roots.

5. Sum the series

$$\frac{1}{\sqrt{2}(1+\sqrt{2})}+\frac{1}{(1+\sqrt{2})\,(2+\sqrt{2})}+$$
$$\frac{1}{(2+\sqrt{2})\,(3+\sqrt{2})}+\text{\&c. } \textit{in infinitum}.$$

6. Prove that if $2\cos. A = x+\frac{1}{x}$, then $2\cos. mA = x^m+\frac{1}{x^m}$.

7. Explain the method of determining the height of an inaccessible object; give the formulæ of solution of the triangles and adapt them to logarithmic computation.

8. The lines drawn from the angles of a triangle, to the bisections of the opposite sides, all meet in one point.

9. A body descends 400 feet down a plane inclined at an angle of 30^0; calculate the actual time of descent to 3 places of decimals.

10. If W be the weight sustained by the wheels of a carriage, what is the force necessary to keep it at rest, upon a road inclined at a given angle to the horizon, the line of draught being parallel to the road?

11. Explain fully the construction and principle of the common pump.

12. The periodic times of the planetary bodies are independent of the eccentricities of their orbits.

13. Explain the phases of Venus.

14. What is the cause of twilight? Within what limits of polar distance, is there at least one day of the year, when it will continue all night?

15. When parallel rays are incident nearly perpendicularly upon a spherical refracting surface, find the geometrical focus of refracted rays.

16. Investigate the rule for finding the *maxima* and *minima* values of a function of one variable, and shew in what manner they are distinguished from each other.

17. Find an expression for the radius of curvature of the ellipse.

18. Find the centre of gravity of the arc of a cycloid.

19. In the collision of perfectly elastic bodies the relative velocity is the same before and after impact.

20. Given the weight of a body in water and in air, to find its true weight.

21. Compare the forces by which the moon is attracted by the earth and sun.

Monday Afternoon.—Mr. Gwatkin.

THIRD AND FOURTH CLASSES.

1. Extract the square root of

$$\frac{a^2 c}{b} - cf + 2ac\sqrt{-\frac{f}{b}}.$$

2. Solve the equation

$$\frac{\sqrt{(a+bx^n)} - \sqrt{a}}{\sqrt{(bx^n)}} = c;$$

and find x and y from the following

$$\left.\begin{array}{l} x^4 - x^2 + y^4 - y^2 = 84 \\ x^2 + x^2 y^2 + y^2 = 49 \end{array}\right\}.$$

3. Produce a given straight line so that the rectangle under the given line, and the whole line produced may equal the square of the part produced.

4. Find by the method of continued fractions a series of fractions converging to $\sqrt{3}$.

5. Prove that the third term of the equation $x^3 - px^2 + qx - r = 0$, cannot be taken away if p^2 be less than $3q$.

6. P and Q sustain each other on two inclined planes, which have a common altitude by means of a string parallel to the planes. Shew from geometrical as well as mechanical considerations that if they be put in motion, their centre of gravity describes a right line parallel to the horizon.

7. Bisect the arc of a semi-cycloid; and if a body oscillate through it, compare the times of describing the first and last half.

8. A right cone whose axis is vertical is just immersed in a fluid, first with its base, then with its vertex downward. Compare the pressure on its whole surface in each case.

9. An object being placed between two plane reflectors inclined at the angle $22^{\circ}.\ 30'$, find the number of images, and shew that two of them coincide.

10. The whole disk of the moon is faintly visible when she is near conjunction, and also when suffering a total eclipse. Explain these phenomena.

11. Find the fluxion of arc whose tang. $= \sqrt{\dfrac{1-x}{1+x}}$, and shew that $\int dx\,(1-x^2)^{\frac{2n-1}{2}}$ (taken between the limits of $x = 0$ and $x = 1$) $=$

$$\frac{1.3.5\ldots.(2n-1)}{2.4.6\ldots.2n}\cdot\frac{\pi}{2}$$

12. Find the area of the curve traced out by the intersection of the sine of an arc, and the secant of half the arc, while the arc increases from 0 to a quadrant.

13. Shew that the number of primes is infinite.

14. Find the polar equation to the ellipse, the centre being considered the pole.

15. Supposing the density of the air to vary as the compressing force, and gravity inversely as $(\text{dist.})^2$ from the earth's center; find the density at any altitude, and shew from the result that the first of the above hypotheses is inadmissible.

Monday Evening.—Mr. GWATKIN.

1. Extract the square root of $14+8\sqrt{3}$.

2. Given the first and last terms, and the sum of an arithmetic series, find the common difference.

3. If three straight lines not in the same plane are equal and parallel, shew that the triangles formed by joining their adjacent extremities are equal and their planes parallel.

4. Shew, that the convex surface of a spherical segment is equal to the area of a circle whose radius is the distance from the pole to the circumference of its base.

5. The bodies A, B, C are acted on in parallel directions by the accelerating forces a, b, c; find the point on which, if connected, they would balance.

6. Define a mean solar year, an apparent solar year, an anomalistic year, and a sidereal year. Explain whence arises the difference between the two first, and write down the three last in order of their length.

7. With a single die, find the chance of throwing the six faces in six trials.

8. Given the base of a triangle, and the exterior angle always equal to three times the interior and opposite angle at the base, required the area of the curve which is the locus of the vertex.

9. Find the principal focus of a concavo convex lens of inconsiderable thickness.

10. If a hemispheroid and a paraboloid have the same base and altitude, shew that their solid contents are as 4 : 3.

11. A paraboloid of given dimensions and specific gravity floats with its axis vertical on a fluid whose specific gravity is known. How far may the axis be increased before it tends to fall from its vertical position.

12. If the difference of two numbers be invariable, shew that as those numbers increase the difference of their logarithms diminishes.

13. Integrate the quantities,

$$\frac{dx}{(bx+cx^2)^2}, \quad \cos.^2 x . e^x \, dx, \quad \frac{dx}{\sqrt{a+x-\sqrt{(a^2+x^2)}}};$$

and shew that $\int \frac{dx}{a+b\cos. x} = \frac{1}{\sqrt{(a^2-b^2)}}$. arc cos. $\frac{b+a.\cos.x}{a+b.\cos.x}$, a being greater than b.

14. Two planes equal in length are inclined at

45^0 and 30^0 to the horizon. A body is projected downward from the top of the first, and another upward from the bottom of the second, each with the velocity acquired down a vertical line equal in length to either plane. Compare the times of describing each plane, and the velocities at the end of the motion.

15. Shew that Newton's trochoid in the sixth section has a point of contrary flexure, and find its position.

16. Find the length of the meridian for any latitude in Mercator's chart, the oblate figure of the earth being considered.

17. Prove that, in the orbit described round the sun by the centre of gravity of the earth and moon, the elliptical form and the equable description of areas are much more nearly preserved than in that which the earth itself describes.

18. Newton, Sect. 9. Prop. 44. Find the ultimate intersection of Cp the radius vector of the moveable orbit and of the line mn which measures the differential force.

19. Integrate the equations,

$$\sqrt{x}.dy = \sqrt{y}.dx + \sqrt{y}.dy;$$

$$x\frac{dz}{dx} + y\frac{dz}{dy} = n\sqrt{(x^2+y^2)}.$$

20. Define the circle of curvature, and *thence* deduce the expressions for its radius and co-ordinates

of the centre. Determine whether the circle of curvature cuts the curve at the point of contact or merely touches it; and apply your result to the case of the ellipse at any point and at the extremities of the semi-axes.

21. The earth being supposed spherical and all its matter collected in the surface, in which a circular aperture of given radius is made, and from whose middle point a body being let fall descends to the centre of the earth, find the velocity acquired at any point of the descent.

22. Explain what is meant by the particular solution of a differential equation, and how it arises. Give the method of deducing it, first from the complete integral, and next from the differential equation; and shew that the results thus obtained coincide.

23. Point out the method of determining the max. and min. values of an expression containing two variables; and give the criterion which decides whether the value thus obtained is a maximum, a minimum, or neither.

24. Shew that the planes of the circles which measure the greatest and least curvature of a surface at any point are at right angles to each other; and having given the radii of these, determine the radius of curvature in a plane which is inclined at any angle to the former.

Tuesday Morning.—Mr. Gwatkin.

FIRST AND SECOND CLASSES.

1. Find the price of a marble slab 5ft. 7in. long, and 3ft. 5in. wide, at 6*s*. per square foot.

2. Construct a tetrahedron upon a given straight line, and find the radius of the sphere described about it.

3. A fraction in its lowest terms whose denominator is prime to 10 produces a circulating decimal. Required proof.

4. Find the right line of quickest descent from a right line to a point, the latter line and point being given in position, but not in the same vertical plane.

5. Shew how the focus of a given parabola may be found.

6. Find the weight and magnitude of a solid by weighing it in two fluids whose specific gravities are known.

7. A small rectilineal object is placed before a spherical reflector at a given distance from it and inclined at a given angle to the axis. Required the position and inclination of the image.

8. Given the base of a triangle and ratio of the angles at the base, draw an asymptote to the curve traced out by the vertex.

9. Integrate the following expressions:
$\frac{\sqrt[3]{(1-x^3)}}{x^5}.dx$, $\frac{dx}{\sqrt{(A+Bx+Cx^2)}}$; and solve the equation $x^2 d^2 y = ay dx^2$.

10. Force $\propto \frac{1}{(dist.)^2}$; shew, that if a particle of matter be attracted to a straight line, the direction in which it begins to move is determined by bisecting the angle formed by the lines which join the particle and the extremities of the attracting line.

11. In the expansion of $(1+x+x^2)^n$ write down the coefficient of x^n.

12. Find the centre of gyration of a cube revolving round an axis which passes through its centre of gravity.

13. Sum the series $\tan. A + \frac{1}{2}.\tan. \frac{1}{2} A + \frac{1}{4} \tan. \frac{1}{4} A + \&c.$ ad infin.

14. Shew how a plane may be drawn touching the surface of any solid; and draw a plane touching in a given point the surface of an ellipsoid whose equation is $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$; x, y, z, being the co-ordinates, and a, b, c, the semi-axes.

Tuesday Afternoon.—Mr. Gwatkin.

FIFTH AND SIXTH CLASSES.

1. Extract the square root of $x^4 - 2x^3 + \frac{3}{2}x^2 - \frac{1}{2}x + \frac{1}{16}$.

2. Solve the equation,

$$\frac{a}{x} + \frac{\sqrt{(a^2 - x^2)}}{x} = \frac{x}{b},$$

and find the values of x and y in the following equations $\qquad x^m\,y^n = a, \quad x^p\,y^q = b.$

3. Draw through a given point a straight line making a given angle with a given straight line.

4. A straight line can cut a circle in only two points. Required proof.

5. Trace the changes of algebraic sign, in the sine of an arc, the tangent and secant; and explain why sec. A and sec. $(180^0 + A)$ which coincide should be one positive and the other negative.

6. In the direct impact of a row of perfectly elastic bodies A, B, C, &c. decreasing in magnitude, shew that the momentum communicated to each is less than that communicated to the preceding body. When is the impact of two bodies said to be direct?

7. Shew that the time in which a heavy body descends down the straight line drawn from any point in the surface of a sphere to the lowest point = the time of descent down the vertical axis of the sphere.

8. A straight line is immersed vertically in a fluid. Divide it into three portions that shall be equally pressed.

9. A straight line passes through the principal focus of a spherical reflector at right angles to the axis. Determine the conic section that forms the image. Where must the straight line be placed that its image may be a circle?

10. Given an ellipse, shew how its centre may be found.

11. $y^3 = ax^2 + x^3$. Trace out the curve. Draw an asymptote to it, and find the magnitude and position of the greatest ordinate.

12. Find the fluxion of the log. of $\frac{x}{\sqrt{(1+x^2)}}$, and of an arc whose sine $= 2x\sqrt{(1-x^2)}$.

13. Integrate the following expressions:

$$\frac{x^4\,dx}{x^2+a^2},\quad \frac{x^4\,dx}{(1-x^2)^{\frac{1}{2}}},\quad \text{and}\quad \frac{dx}{(x-a)^2.(x-b)}.$$

14. Describe the transit instrument and adjust it to the plane of the meridian.

15. Find the center of gravity of a spherical sector.

16. Two bodies fall to a center of force from the same distance, one acted on by a force varying as the distance, and the other by a force $\propto \frac{1}{(\text{dist.})^2}$. The forces at first being supposed equal, compare the times of descent.

17. Given the velocity, distance, and direction of projection, when the force varies as the distance, shew that the body describes an ellipse; and find the magnitude and position of its semi-axes.

Tuesday Afternoon.—Mr. Peacock.

THIRD AND FOURTH CLASSES.

1. If the roots of the equation

$$x^3 - px + q = 0$$

be real, and if we assume $\cos.\theta = \frac{-q}{2}\sqrt{\frac{27}{p^3}}$, then one of the roots $= 2\cos.\frac{\theta}{3}.\sqrt{\frac{p}{3}}$.

2. Determine the conjugate diameters of an ellipse, which make the least angle with each other.

3. The radius of curvature is a tangent to the evolute.

4. Investigate a general expression for the coordinates of the centre of gravity of the area of a curve, included between a given ordinate and abscissa.

5. Given the quantities and directions of three forces acting upon a material point in different planes, to determine the quantity and direction of the resultant or compound force.

6. In the interior rainbow, the tangent of the angle of incidence is twice that of the angle of refraction.

7. A sphere of less specific gravity than water, ascends from the depth a; what is its velocity at the moment it reaches the surface?

8. Explain the method of determining the heliocentric latitude and longitude of a planet.

9. Enumerate the principal phænomena of Saturn's ring.

10. Find the centre of oscillation of a cylinder of given length and diameter, suspended by its extremity.

11. Prove, that tan. $nA=$

$$\frac{n \tan. A - \frac{n(n-1)(n-2)}{1.2.3}(\tan. A)^3 + \&c.}{1 - \frac{n(n-1)}{1.2}(\tan. A)^2 + \&c.}$$

12. Find the whole area of the curve whose equation is $a^2y^2 - a^2x^2 + x^4 = 0.$

13. Find the locus of the points, in the plane of the moon's orbit, where a body will be equally attracted by the earth and moon.

Tuesday Evening.—Mr. Peacock.

1. If two spherical triangles have two sides of one triangle equal to two sides of the other, each to each, and the included angles equal, the triangles are equal in every respect.

2. The modulus of tabular logarithms or

$$M = .4342944819;$$

shew in what manner this number is determined.

3. It is always possible to find those roots of numerical equations, which are whole numbers or rational fractions, without the aid of formulæ of approximation.

4. Explain the method of determining the position of the nodes of the moon's orbit: what is the physical cause of their retrograde motion?

5. The friction of a body being supposed independent of velocity, to find an expression for the time of a body's descent down a given inclined plane, the friction being equal to $\frac{1}{n}$th part of the pressure.

6. A cubical iceberg is 100 feet above the level of the sea, its sides being vertical: given the specific gravity of sea water $= 1.0263$ and of ice $= .9214$, at the temperature of 32^0, to find its dimensions. Is this position one of stable equilibrium?

7. Prove that the centres of oscillation and suspension are reciprocal. Of what use is this property, in the determination of the length of a pendulum which vibrates seconds in any given latitude?

8. Explain the method of determining the ratio of the sines of incidence and refraction both in liquid and solid bodies.

9. Given the latitudes and longitudes of two places, where the inclination of the magnetic needle is nothing, to find the point of the terrestrial equator, which is cut by the magnetic equator, supposing it a great circle of the earth.

10. Of all equal quadrilateral figures, the square has the least perimeter.

11. Integrate

(1.) $\dfrac{dx}{x\sqrt{(bx^2-a)}}$, and $\dfrac{d\theta}{(\sin.\ \theta)^4 \cos.\ \theta}$.

(2.) $\dfrac{dx}{\sqrt{(a^4-x^4)}}$ from $x=0$, to $=a$.

(3.) $\dfrac{\left(1+\dfrac{dy^2}{dx^2}\right)^{\frac{3}{2}}}{\dfrac{d^2y}{dx^2}} = \dfrac{a^2}{2x}$.

(4.) $\dfrac{dx}{\sqrt{(1-x^2)}} + \dfrac{dy}{\sqrt{(1-y^2)}} = 0$.

(5.) $xy - \dfrac{d^2z}{dx\,dy} = 0$.

12. Find the equation of the curve which is the locus of the extremities of the perpendiculars from the centre upon the tangents of the equilateral hyperbola, and determine the position of its tangents at the points where it cuts the axis.

13. Given

$$\begin{aligned} \log. 510 &= 2.70757018 \\ \log. 511 &= 2.70842090 \\ \log. 513 &= 2.71011737 \\ \log. 514 &= 2.71096312 \end{aligned}$$

to find the logarithm of 512, by the method of interpolations.

14. Explain the principle and construction of the achromatic telescope.

15. What is the least velocity with which a body must be projected from the moon, in the direction of a line joining the centres of the earth and moon, so that it may reach the earth?

16. If the bulb of a thermometer be a sphere, whose diameter is 1 inch, and if the diameter of the tube be $\frac{1}{10}$th of an inch, what is the pressure upon the interior of the bulb, when the mercury stands at the altitude of 10 inches above it, exclusive of that portion of the pressure which sustains the mercury in the tube?

17. If $nt = u + e \sin. u$, where u is the eccentric and nt the mean anomaly, apply *Lagrange's* Theorem to the developement of $a(1 - e \cos. u)$, in terms of cosines of nt and its multiples.

18. Prove, that in going from the equator to the pole, the increment of gravity varies very nearly as the square of the sine of the latitude. In what manner does this variation affect, 1st, the length of a pendulum vibrating seconds, and 2dly, the altitude of the barometrical column?

19. Prove, that there can be no more than five regular solids; and find the angles which their terminating planes make with each other.

20. Given the weight of the key-stone of a circular arch, in a state of perfect equilibration, and the angles formed by each of its faces with a vertical line; to find the horizontal pressure upon the abutments.

21. Prove, that

$$\tan.^{-1}\frac{x}{y} = \tan.^{-1}\frac{ex-y}{ey+x} + \tan.^{-1}\frac{e_1-e}{ee_1+1} + \tan.^{-1}\frac{e_2-e_1}{e_1e_2+1} + \ldots\ldots\ldots + \tan.^{-1}\frac{e_n-e_{n-1}}{e_{n-1}e_n+1} + \tan.^{-1}\frac{1}{e_n};$$

where $\tan.^{-1}\frac{x}{y}$ represents an arc whose tangent is $\frac{x}{y}$, and where $e, e_1, e_2, \ldots e_n$ are any numbers whatever.

22. A spherical shell with a small orifice at it's lowest point, is filled with air of the density of the atmosphere, and immersed in water to a depth a: with what velocity will the water rush into the shell, and what portion of the sphere will it occupy, when the motion ceases?

23. Develope $\frac{x}{e^x - 1}$ in a series involving ascending powers of x. Of what use are the coefficients of this series in expressing the law of the coefficients of the series for tan. θ in terms of θ?

24. Enumerate, as Newton has done, the principal proofs of the truth of the theory of universal gravitation.

1820.

Monday Morning.—Mr. WHEWELL.

MONDAY, JANUARY 17, 1820.

FIRST AND SECOND CLASSES.

1. GIVEN two sides and the included angle, find an expression for the area, (1) in a plane, and (2) in a spherical triangle.

2. A straight line cuts a parabola, whose vertex is A, in two points P and Q, and its axis in O; ordinates PM, QN, being drawn, shew that AO is a mean proportional between AM and AN.

3. The force varies inversely as $(\text{distance})^{\frac{10}{3}}$. A body is projected from any point in any direction, with a velocity equal to that from infinity. Find the position of the apse, and the whole angle described.

4. On a horizontal dial the angle corresponding to a second of time at 4 o'clock, is double the angle for a second at noon. Find the latitude of the place.

5. The equation to a curve is $y=x\sqrt{\frac{a^2-x^2}{a^2+x^2}}$. Trace it; find its maximum ordinate, and its area.

6. The earth revolving round a fixed axis, shew that a body let fall from the top of a high tower will not strike the ground exactly at the foot of the tower. Between what cardinal points of the compass will the point struck be situated with respect to the foot of the tower?

7. Express the distance of a point from the earth's center in terms of the latitude.

8. A point T moves uniformly along a straight line; another point P, with *three* times the velocity, always moves towards T, so as to describe the *curve of pursuit*. Trace the curve, and shew that the path described by T from the time when the paths are at right angles till it is overtaken by P is $\frac{3}{8}$ of their distance at that time.

9. The equation to the elliptical paraboloid being $ax^2+by^2+abz=abc$, draw a normal to it; and determine the points where this line cuts the three co-ordinate planes. Also find the solid content of a portion contained by planes parallel to the planes of xz and yz.

10. Find right-angled triangles, such that all the sides shall be rational numbers.

11. If a, b, c be the sides of a plane triangle and

C the angle opposite to c; prove that

$$\text{hyp. log. } c = \text{hyp. log. } a - \frac{b}{a}\cos. C - \frac{b^2}{2a^2}\cos. 2C - \frac{b^3}{3a^3}\cos. 3C - \&c.$$

12. Integrate the differential

$$dx\sqrt{\frac{1-e^2x^2}{1-x^2}},$$

in a series which converges rapidly when e is nearly $=1$; and the equation

$$(a+y)\frac{dx}{dy} = x + y - x\frac{dy}{dx}.$$

13. Define the moon's *variation*. Give Newton's construction for it, and hence shew how it varies.

Monday Afternoon.—Mr. Whewell.

FIFTH AND SIXTH CLASSES.

1. Find the value of $.151\dot{6}3\dot{6}363$, &c. of £1.

2. Find in what time at compound interest, at 5 per cent. a sum will become 10 times its original value. (N. B. the log. of 105 is 2.0211893.)

3. Solve the equations

$$x + \sqrt{\{x^2 + \sqrt{(x^2+96)}\}} = 11$$

$$\left.\begin{array}{l} x(y+z) = a \\ y(x+z) = b \\ z(x+y) = c \end{array}\right\}$$

$x^3 - 6x - 40 = 0$, by Cardan's method.

$$3^{1x} . 2^{3x} = 10.$$

4. A beam rests with one end on a horizontal plane, and the other against a vertical wall; find the horizontal force necessary to prevent its lower end from sliding outwards.

5. A projectile is to be thrown across a plain 120 feet wide, to strike a mark 30 feet high, the velocity of projection being that acquired down 80 feet; find at what angle it must be projected.

6. A piece of wood weighs 12 lbs. and when annexed to 22 lbs. of lead, and immersed in water, the whole weighs 8 lbs. The specific gravity of lead being 11 times that of water, find the specific gravity of the wood.

7. A cylinder whose axis is horizontal empties itself by a hole in the lowest part; find the time.

8. A trapezium has two opposite sides equal, and the other two parallel; compare the resistance upon it, when it moves in the direction of the parallel sides, and when it moves in a direction perpendicular to them.

9. Explain why all parts of the field of view of a telescope are not equally bright; and find the proportion of the bright part to the whole in the astronomical telescope.

10. Having observed the elongation of a planet when stationary, shew how its distance from the sun may be found.

11. Find the integrals of

$$\frac{dx}{\sqrt{a}-\sqrt{x}}, \quad \frac{x\,dx}{\sqrt{(2ax-x^2)}}, \quad dx.\log. x.$$

12. Find an expression to determine the force by which a body may be retained in a given curve: apply it to the curve whose equation is

$$p^2=\frac{b^2\,x^4}{a^4+x^4}.$$

13. The force varying inversely as the distance, find the angle between the apsides in an orbit nearly circular, and prove the method.

14. Sum the following series to n terms,

$$\frac{1}{\sqrt{x}}-\frac{\sqrt{2}}{x}+\frac{2}{x\sqrt{x}}-\&c.$$

and continue the harmonic progression.... 3, 4, 6, ... upwards and downwards. How far can it be continued either way?

15. Form the equation, whose roots are

$$2+\sqrt{-3},\ 2-\sqrt{-3},\ 1, \text{ and } -5.$$

16. In any plane triangle of which the sides are a, b, c, and the opposite angles A, B, C, prove that

$$\sin. A=\frac{a.\sin. C}{\sqrt{(a^2-2ab.\cos. C+b^2)}}.$$

17. How high will a given balloon ascend? When it floats in the air, supposing that a given weight of ballast is thrown out, to what additional height will it rise, and how much will a barometer in it sink?

Monday Afternoon.—Mr. Wilkinson.

THIRD AND FOURTH CLASSES.

1. Find the discount of £.100 for one year at 5 *per cent.* and then calculate the interest on this discount for the same time.

2. Solve the equations

$$\frac{x^4+1}{(x+1)^4} = \frac{1}{2}, \text{ and } \sqrt{(x^4-1)}+\sqrt{(x^2-1)}=x^3.$$

3. If a rigid sphere, revolving round an axis, become fluid and therefore change its figure, the whole moment of inertia will remain the same as before.

4. Find the centers of oscillation of a square suspended by one corner and oscillating *flatways* and *edgeways*.

5. What is the reason that waves always break against the shore whatever be the direction of the wind?

6. The horizon of any place being taken as the plane of projection, find the figure and dimensions of the path of the diurnal motion of a given star orthographically projected.

7. What is the meaning of the astronomical term equation? The equation of time (arising from what causes?) is a maximum about the beginning of November, is it then additive or subtractive?

8. The number of impossible roots of any equation $x^n - px^{n-1} + \&c. = 0$ is not increased by multiplying its terms by the successive terms of the series 0, 1, 2, 3, 4, &c.

9. Integrate these expressions

$$\frac{dz}{1-\tan.^2 z}, \text{ and } \frac{dx}{1+x+x^2}.$$

10. If a tangent be drawn at the extremity of the latus rectum of a conic section meeting the tangent at the vertex, the part of this latter tangent thus cut off shall be equal to the distance between the vertex and the focus of the curve.

11. Find in what curve a body must revolve round a repulsive force varying as the distance from a point, so that its velocity may always equal that in a circle at the same distance, round an equal attractive center of force.

12. Transform the equation to the lemniscata $(x^2+y^2)^2 = a^2.(x^2-y^2)$, from rectangular to polar co-ordinates.

13. If in an equation between x and y the sum of the indices be the same in every term, the loci of the corresponding values of x and y are straight lines.—Find what lines are defined by the equation $y^3 - 2xy^2 + x^3 = 0$.

Monday Evening.—Mr. WILKINSON.

1. What is the purchase of £1034. 15*s*. stock in the 3 per cents. at $62\frac{1}{2}$?

2. If two straight lines in space be parallel, their projections on any plane will be parallel.

3. Find the solidity of an octahedron.

4. Required the present worth of an annuity of (a) pounds for n years payable every instant in equal portions, interest also being convertible into principal as fast as it becomes due.

5. R and r being the radii of the circumscribed and inscribed circles of the triangle whose sides are a, b, c, shew that

$$Rr = \frac{abc}{2(a+b+c)}.$$

6. If n be a prime number and a and b integers not divisible by n, then $\frac{a^{n-1} - b^{n-1}}{n}$ is a whole number.

7. State D'Alembert's principle, and the principle of virtual velocities, and employ them in deducing this theorem $4m(Ax + By + Cz) = Aa^2 + Bb^2 + Cc^2$; A, B, C, being *weights* which put a system in motion, x, y, z, the spaces perpendicularly descended by them respectively, and a, b, c, their actual velocities.

8. A slender rod of uniform thickness revolves round an axis passing through one of its extremities; find,

(1.) At what point an obstacle must be opposed to it that there may be no stress on the axis from the shock?

(2.) What quantity of matter should be collected in this point that the impulse on the obstacle may be the same as that of the rod?

(3.) At what distance from the axis the obstacle must be opposed that the impulse may be the same as if the whole matter in the rod were collected in that point?

9. The earth being supposed a sphere of uniform density, shew that the pressure on each side of a plane passing through its center : the whole weight of the earth :: 3 : 16.

10. A piston is thrust down uniformly into a cylinder full of air, having a small orifice at the end; find the quantity discharged in a given time into a vacuum.

11. If parallel rays fall upon a spherical refracting surface, the distance from the axis of the geometrical focus of a small pencil which does not pass through the center, is proportional to the cube of the distance at which it is incident from the axis.

12. The sun's right ascension on two successive days at noon was $6^h. 40'. 25''$, and $6^h. 45'. 13''$, by

the Nautical Almanack (and therefore in sidereal time); the moon's right ascension at the same time was from the Nautical Almanack (and therefore expressed in degrees) $5^s. 9^0. 32'$, and $5^s. 20^0. 9'$. Required the time of the moon's transit in the interval.

13. The length of a degree perpendicular to the meridian is always greater than the degree of the meridian corresponding.

14. Define the axis of a curve, and draw the axis to the curve of which $y^3 - 3axy + x^3 = 0$ is an equation.

15. T and t are the parts of the tangent at the vertex (A) of a rectangular hyperbola, (whose semi-axis $= 1$) cut off by lines (CP, Cp) drawn from the center to the curve; shew that if the sector CAP be n times the sector CAp,

$$\frac{1-T}{1+T} = \left(\frac{1-t}{1+t}\right)^n.$$

16. A's skill is double B's, and their stakes equal; find what C, whose skill is equal to A's, must stake, that A's advantage may be as great as if he played with B.

17. If it be an even wager that D wins (n) successive games of E, what is E's chance of winning the first game?

18. Integrate the differential equations

$$\frac{dy}{dx} = a \sin. x + by; \text{ and } \frac{d^2 y}{d x^2} - \frac{2dy}{dx} + 2y = 0;$$

and shew how to separate the variable quantities in

$$\frac{dy}{dx} = \frac{a^2 + x^2 - y^2}{a^2}.$$

19. Apply the method of increments to sum the series

$\frac{3}{1.2} \cdot \frac{1}{2} \cdot \frac{4}{2.3} + \frac{1}{2^2} + \frac{5}{3.4} \cdot \frac{1}{2^3} +$ &c. (to n terms), and also to shew that sin. x + sin. $3x$ + sin. $5x$ + ... (to n terms) $= \frac{\sin.^2 nx}{\sin. x}$.

20. If tangents be drawn at the extremities of the major axis of an ellipse, the rectangle under the parts of these tangents intercepted by the tangent at any other point of the curve is a constant quantity.

21. Conversely, if AT, Vt, be two perpendiculars to a given line AV and on the same side of it, and also the rectangle $AT \times Vt$ be constant; to investigate the nature of the curve which the line Tt perpetually touches.

22. The axes of two equal cylinders intersect each other at right angles, find the content of the solid cut out of each by the surface of the other.

23. The attractions of *ellipsoids* on particles in the surface in directions perpendicular to any of the principal sections are as the distances.

24. Two bodies connected by an inflexible rod

without weight, and to one of which a certain velocity is communicated, are constrained to move along two grooves at right angles to each other; required the circumstances of motion; and shew that when the bodies are equal the line which joins them revolves uniformly.

Tuesday Morning.—Mr. Wilkinson.

FIRST AND SECOND CLASSES.

1. What will the ceiling of a room come to, whose length is 24ft. 6 in. and width 16ft. 3in. at 3*s*. per yard?

2. $u = \text{arc}\left(\tan.\frac{x-y}{x+y}\right)$ find the differential of u.

3. Every positive number has an infinite number of logarithms, only one of which is real, and every negative number has only imaginary logarithms.

4. When a system of bodies connected in any manner is in equilibrium the center of gravity is as high or as low as is possible.

5. The resistance on a cube moving in a fluid in the direction of its diagonal : resistance on the same cube moving in a direction perpendicular to its side :: $1 : \sqrt{3}$.

6. A conical vessel is filled with water; find that

heavy sphere which when put into it shall force out the greatest quantity of fluid.

7. Draw a tangent to the curve formed by the intersection of a right cone with the cylinder erected on the radius of the base as a diameter.

8. Investigate the nature of the curve trisecting all the arcs described on the same chord.

9. Integrate

$$\frac{ydx - xdy}{(x+y)^2}, \text{ and } xdy - ydx = \frac{2xdy - ydx}{\sqrt{(x^2+y^2)}}.$$

10. *A*, and *B* whose skill is *m* times *A*'s, agree to play with this law, that *A* shall continue to stake against *B* so long as *B* wins without interruption; shew that *B*'s expectation is worth $(m-1)$ times the stake.

11. A circle of curvature is described at the vertex of a parabola, and another circle which touches that and both the arcs of the curve, and so on continually; compare the sum of all the areas of these circles with that of the parabola.

12. If D and D' be the lengths of a degree of a meridian at the equator and in latitude λ respectively, a and b the equatorial and polar diameters,

$$\frac{a}{b} = \frac{\sin. \lambda}{\sqrt{\left(\frac{D}{D'}\right)^{\frac{3}{2}} - \cos.^2 \lambda}}.$$

13. Explain how the comparative densities of the

sun and moon have been deduced from the phænomena of precession and nutation.

Tuesday Afternoon.—Mr. Wilkinson.

FIFTH AND SIXTH CLASSES.

1. Required the value of 1cwt. 2qrs. 3lbs. at £4. 5*s*. 6*d*. per cwt.

2. Find the value of

$$\frac{x-x^5}{1-x^2} \text{ when } x=1.$$

3. Write down the 4 first terms of the expansion of

$$\left(x-\frac{1}{x}\right)^{-\frac{1}{2}}.$$

4. The two bases of any oblique prism are reciprocally proportional to the sines of the angles which they make with the axis.

5. Given the forces of many agents, to find the time in which they will all produce a given effect.

6. Find the time of one oscillation in a cycloid.

7. A row of non-elastic balls whose magnitudes increase in geometrical progression are placed at equal distances in a straight line, and a given velocity is communicated to the first; required the time before the n^{th} is put in motion.

8. Explain the method of finding the specific gravity of a body lighter than water.

9. Construct Newton's telescope, and shew that objects appear inverted through it.

10. The aberration of a given star in right ascension is not necessarily nothing when that in declination is a maximum.

11. Integrate

$$\frac{dx}{x^2+1}, \quad \frac{dx}{\sqrt{(x^2+1)}}, \text{ and } \frac{dx}{x\sqrt{(x^2+1)}}.$$

12. Find the area and point of contrary flexure of the curve, whose abscissa is always equal to the arc of a circle, the versed sine of which is the ordinate.

13. If $S = 1 + \frac{1}{2^m} + \frac{1}{3^m} + \frac{1}{4^m} +$ &c. *in inf.*

and $s = 1 - \frac{1}{2^m} + \frac{1}{3^m} - \frac{1}{4^m} +$ &c. *in inf.*

Shew that $S : s :: 2^{m-1} : 2^{m-1} - 1$.

14. The total number of odd combinations that can be formed out of any number of things is greater by unity than the total number of even.

15. If at any point P in an ellipse the ordinate NP be produced to meet the tangent at the extremity of the latus rectum, the whole line thus produced is equal to SP the focal distance.

16. The force to a plane must vary inversely as the cube of the distance in order that the trajectory may be a semi-circle.

17. If two bodies describe about each other and about their common center of gravity similar and concentric ellipses, the forces with which they attract each other are proportional to their distances.

Tuesday Afternoon.—Mr. WHEWELL.

THIRD AND FOURTH CLASSES.

1. In the common system of notation explain why the number of digits cannot be more or less than the local value 10.

2. To find a point within a given triangle from which the three sides shall subtend equal angles.

3. A body is projected with the velocity due to a height h, at an angle A with the horizon. Find an expression for the latus rectum of the parabola described.

4. Shew how the points of contrary flexure in spirals may be found; and apply the method to find the point of flexure of the *lituus* whose equation is $\theta = \frac{a^2}{r^2}$, θ being the angle, and r the radius vector.

5. Shew that a body cannot move so that the

velocity shall vary as the space from the beginning of motion. And if the velocity vary as the cube root of the space, find how the time and the force vary.

6. Two bodies P and Q, whose specific gravities are m and n, balance each other on a given straight lever. When the whole is immersed in water whose specific gravity is 1, what alteration must be made in the place of the fulcrum, that they may continue to balance?

7. If a right cone, the diameter of whose base is BC, and vertex A, be cut by a plane so that the section may be an ellipse whose major axis is PQ, the solid content of the part towards the vertex is to that of the whole, as $(APQ)^{\frac{3}{2}}$ to $(ABC)^{\frac{3}{2}}$; APQ and ABC being the areas of those triangles. Also find the equation to the surface of a cone referred to three rectangular co-ordinates.

8. A beam hangs, by means of a given cord fastened to its upper end, from a fixed point in a vertical wall. Against what point of the wall must its lower end be placed, that it may have no tendency either to ascend or descend? Within what limits for the length of the beam is this equilibrium possible?

9. The equation $x^4 - 45x^2 - 40x + 84 = 0$, has two roots whose difference is 3; find them.

10. The force varies inversely as the square of the distance. A body is projected in a direction which

makes an angle of 60^0 with the distance, with a velocity which is to the velocity from infinity as $1 : \sqrt{3}$. Find the major axis, the position of the apse, and the excentricity, of the ellipse which will be described; and the periodic time.

11. Integrate

$$\frac{dx}{(1-x^3)^{\frac{1}{3}}};\ e^{\sqrt{x}}xdx;\ \sin. mx.\sin. nx.\cos. px.dx;$$

$$\frac{dy}{dx} = \frac{a+2x-y}{a-x+2y}.$$

12. Sum the series

$$\frac{1}{1.3} - \frac{1}{2.4} + \frac{1}{3.5} - \&c. \textit{ in inf.}$$

$$\frac{2}{1.3.7} + \frac{4}{3.7.15} + \frac{8}{7.15.31} + \&c. \text{ to } n \text{ terms}$$

by increments:

and prove $1^2 + n^2 + \left(\frac{n(n-1)}{1.2}\right)^2 +$

$$\left(\frac{n(n-1).(n-2)}{1.2.3}\right)^2 + \&c. = \frac{1.2.3\ldots 2n}{(1.2.3\ldots n)^2}.$$

13. If we divide a, a^2, a^3, . . . by a prime number p, (a being any number) we shall obtain a remainder 1 before we have taken p terms. Also after this remainder the remainders will recur.

Tuesday Evening.—Mr. Whewell.

1. The area of any right angled triangle is equal to the rectangle of the semi-perimeter and the excess of the semi-perimeter above the hypotenuse. Required proof.

2. *A* sets off from London to York, and *B* at the same time from York to London: they travel uniformly; *A* reaches York 16 hours, and *B* London 36 hours, after they have met on the road; find in what time each has performed the journey.

3. The surface of a right cone being given, to find its form that the solid content may be the greatest possible.

4. The equation to a straight line being $y = ax + b$, find the equation to another straight line drawn perpendicular to the first and passing through a given point. Also, solve the same problem when the lines are referred to *three* rectangular co-ordinates.

5. Two given spheres are moving in given straight lines with given uniform velocities; find where they will meet, (1) when their directions are in the same plane, (2) when they are not.

6. A slender cylinder, whose specific gravity is $\frac{1}{2}$ that of water, is placed in the fluid in an oblique position: find the magnitude, direction, and point

of application of the force which must act on the cylinder to keep it immersed $\frac{5}{6}$ of its length.

7. A gate of given weight and form is hung by hinges to a post inclined at a given angle from the vertical. When it swings freely, find the time of its small oscillations.

8. The perihelion distance of a comet is $\frac{1}{3}$ the distance of the earth from the sun; and its orbit, which is parabolical, and the earth's, which is circular, are in the same plane: how many days is the comet within the earth's orbit?

9. Describe the *repeating circle*, and the method of observing with it; and explain its advantages.

10. Find the time of a body falling through any space towards a center of force, (1) when the force varies inversely as the square root, (2) when it varies inversely as the cube root, of the distance. For what laws of force is the integration, which gives the time, practicable?

11. The force varies inversely as the fifth power of the distance. A body is projected with a velocity which is to the velocity from infinity as 5 to 3, and in a direction which makes with the line drawn to the center an angle whose sine is $\frac{2\sqrt{6}}{5}$. Find the orbit, and the time of describing a given angle; and determine whether it has an apse.

12. *AM*, *MP*, *MT*, are the abscissa, ordinate,

and subtangent of a curve, of which the property is that $AM : MP :: MT : TA$; find its equation, and trace it.

13. Integrate the following differentials and differential equations,

$$\frac{dx\sqrt{(1-x^2)}}{1+x^2}; \ \epsilon^{ax} \sin.^2 x . dx;$$

$$x^m(y\,dx+x\,dy)=y^n\,(y\,dx-x\,dy);$$

$$d^3y - 3\,d^2y\,dx + 3\,dy\,dx^2 - y\,dx^3 = 0;$$

$$x\frac{dz}{dx}+y\frac{dz}{dy}=z.$$

14. If P and S, attracting each other with forces proportional to their masses, revolve round their center of gravity; their periodic time is to the periodic time of an indefinitely small particle, which describes a similar orbit round S at the same distance, as $\sqrt{S}$ to $\sqrt{(S+P)}$. Also, if to P be annexed an equal mass which is *not attracted* by S, what alteration will be made in this proposition?

15. In any polyhedron the number of solid angles together with the number of plane faces, exceed by 2 the number of *edges*.

16. On two straight lines at right angles to each other, two points move respectively from given positions, with equal uniform velocities; find the curve to which the line which joins them is always a tangent.

17. In a bag are 8 bank notes, viz. 1 of twenty pounds, 2 of five, and 5 of one: a person is allowed

to take out three indiscriminately: what is the value of his expectation?

18. The radiating point and the caustic being given, shew that there are an infinite number of reflecting curves which will produce the caustic.

19. A body falls towards a center of force which varies directly as the distance, in a medium of which the resistance varies as the square of the velocity: find the velocity at any point. Shew from your result that when the resistance vanishes, the velocity coincides with that in a non-resisting medium.

20. Let $yz=\phi\left(\frac{y}{x}\right)$ be the equation to a curve surface, where ϕ is any function whatever: shew that the part which the tangent plane to any point cuts off from the axis of z, is twice the value of z for that point.

21. Sum the series,

$$\frac{1}{1.4}-\frac{1}{3.6}+\frac{1}{5.8}-\text{\&c. in inf.}$$

$$x.\cos. A+x^2.\cos. 2A+x^3.\cos. 3A+\text{\&c. in inf.}$$

Also, prove that

$$\frac{1}{1^2+1}+\frac{2}{2^2+1}+\frac{3}{3^2+1}+\text{\&c. in inf.}$$

$$=\frac{\pi}{2}.\frac{\epsilon^{2\pi}+1}{\epsilon^{2\pi}-1}-\frac{1}{2};$$

and find an expression for the n^{th} term of a series where each term is the product of the two preceding.

22. AB is the axis of a cycloid, (of which A is the vertex,) and C its middle point. An ordinate is drawn meeting the axis in M, the cycloid in P, P', and the generating circle in Q, Q'. CN is taken towards A equal to AM. Then the cycloidal sector PNP' = triangle QBQ'. Required proof.

23. If a chain of given length be suspended from two points, shew that its center of gravity is lowest when its form is a catenary.

24. Expand the radius vector of an ellipse about the focus $\left(=r=\frac{a(1-e^2)}{1+e.\cos.\theta}\right)$ in a series of the form

$$A+B.\cos.\theta+C.\cos.2\theta+D.\cos.3\theta+ \&c.$$

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